More Than Reshuffling

Lessons from an Innovative Remedial Math Program at The City University of New York

a companion paper to

More Than Rules—College Transition Math Teaching for GED Graduates at The City University of New York

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Introduction

More Than Rules was released in September of 2009.[1] The goal of that paper was to describe the challenges facing GED graduates at The City University of New York (CUNY) and the work of the CUNY College Transition Program (CTP) to prepare students for those challenges. Early sections of that paper examined the misalignment between the GED math subject test and college math placement exams, the prevalence of math remediation for GED graduates, and the generally poor educational outcomes for these students at CUNY. The heart of the paper described non-traditional approaches to math teaching and learning in CTP, including content, pedagogy, curriculum, and instructor development. Student outcomes for the fall 2008 and spring 2009 cohorts demonstrated some promising early results.

Just as *More Than Rules* was released, CTP was restructured and became known as the CUNY College Transition Initiative (CTI). The new program moved "across the college wall" and is now offered as an option to freshmen who have taken and failed multiple placement exams. The most important new feature of the CTI math course was the significant increase in the quantity of instruction. That change has made it possible to examine whether an increase in instruction, while maintaining the pedagogical approaches and other features of the CTP course, could help many more students eliminate their need for one or both levels of remedial math while also making progress on a more extensive set of math learning goals.

This paper will review the first year of CTI math. This is a "companion" paper to *More Than Rules* because the CTP approaches to math content, curriculum, instructor development, and pedagogy that were described in that paper also provided the underlying foundation for CTI math. If possible, *More Than Rules* should be read before this paper.[2]

The first section of this paper outlines CTI program structure. This new structure required a significant expansion of the CTP math curriculum, and the second section describes how this was accomplished. The third section includes enrollment and retention information for students at two CTI sites across two semesters. Assessment of student learning is described in the fourth section. In the fifth section, I locate CTI math teaching and learning within the context of more typical mathematics instruction that exists and that is encouraged in standards documents for school and college educators. I also compare CTI math to other common remedial math reform efforts that are underway in community colleges. In the final section, for those who may wish to experiment or commit to CTI-like instructional practices, I outline the institutional and other conditions that need to be in place in order to make this sort of pedagogical change possible.

I would like to thank Kevin Winkler, Rosanne Proga, and Drew Allen for their assistance in gathering data for use in this paper, and John Mogulescu, Leslee Oppenheim, John Garvey, Charlie Brover, Denise Deagan, and Mark Trushkowsky for their comments on early drafts.

The views expressed in this paper are my own.

[1] *More Than Rules: College Transition Math Teaching for GED Graduates at The City University of New York,* by Steve Hinds, CUNY Office of Academic Affairs; 2009.
[2] *More Than Rules* can be located on the web at http://www.cccs.edu/Docs/Foundation/SUN/Math%20Paper.pdf

The CUNY College Transition Initiative

In the summer of 2009, the CUNY College Transition Initiative (CTI) was established by the CUNY Central Office of Academic Affairs in collaboration with two of its campuses—LaGuardia Community College and Kingsborough Community College. CTI was not a new program but a restructured version of the CUNY College Transition Program (CTP) which provided academic preparation and advisement for CUNY-bound GED graduates over the previous two years (and which was described in detail in *More Than Rules).*The shift from CTP to CTI included significant changes in program design, several of which are described below. For a chart that summarizes elements of the different programs, see *Appendix A*.

Student Eligibility

Previously, students joined CTP before they applied to a CUNY college or took any college placement exams. The program was positioned as a semester of study alongside or after GED study but always *before* college entrance. Students now join CTI *after* they have been admitted to a CUNY college, *after* they have taken the college placement exams, and *after* they have failed (at least) the math and writing exams. CTI has moved "across the college wall" and is presented as an alternative to traditional remedial courses once placement test results show that significant remediation is needed. Focusing on students who fail exams in at least two basic skills areas ensures that CTI will serve GED (and some high school) graduates who are considered to be among the academically least-prepared students arriving at participating CUNY colleges.

Instructional Intensity and Duration

Math, reading, and writing instruction in CTI increased in two ways compared to CTP, with more instructional hours per week and a longer semester of study. Weekly math instruction increased from six hours in CTP to nearly twice that amount in CTI. The semester of study changed from fourteen weeks in CTP to a two-part, eighteen-week semester in CTI. In the first twelve weeks—known as "Phase One"—students are placed in learning communities that include math, reading/writing, and group advisement, as was the case in previous CTP semesters.[3] After Phase One, all students re-test in any basic skills exams they needed to pass. Students with strong exam results are then enrolled in a three-credit course at the college over a six-week "Phase Two" at no cost. Students who still have basic skills needs after Phase One testing continue to study in those areas during Phase Two. At the end of Phase Two, students re-take any exams they still need. Therefore, CTI students have two opportunities (if needed) to pass basic skills exams over eighteen weeks of study. For a diagram that displays these pathways through CTI, see *Appendix B.*

All students (whether they need pre-algebra, algebra, or both) take a Phase One math course that blends pre-algebra and algebra content over a total of 130 hours.[4] Students who also need Phase Two math instruction receive somewhere between 35 and 60 hours of additional math instruction, depending on whether they need more study in one or both math levels.

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[3] "Learning community" is only used here to describe a structure where the same 20-25 students are grouped together for math instruction, reading/writing instruction, and weekly academic advisement. The content of the math course is not integrated with the content of the reading/writing course.

[4] To meet the basic skills requirement in mathematics at CUNY community colleges, students must pass placement exams in pre-algebra and algebra exams that are produced by ACT, Inc. and are known as the "COMPASS" exams. During the fall 2009 and spring 2010, CUNY used a scaled score of 30 as the passing standard on each exam. That passing score will change in the spring 2011 semester, and the implications of this will be discussed later in this paper. Students who fail one of the COMPASS math exams will typically need to take and pass one remedial math course; students failing both will typically need to pass two remedial courses. To be eligible for the Phase One CTI course, students may have failed one or both of the exams.

Instructional and Advisement Staff

Before CTI was established, math and reading/writing instructors were paid at part-time rates to teach CTP courses. The instructors needed to combine this work with other teaching in order to reach full-time status. Academic advisement was also treated as a part-time assignment, in some cases done by a CTP instructor and in other cases by the counselor from the Adult Learning Center at that campus.[5]

CTI was established with funding that supports a three-person team entirely devoted to two groups of students at each site—one full-time math instructor, one full-time reading/writing instructor, and one ¾-time academic advisor. Full-time instructors teach 20 hours per week (10 hours for each group of students). In addition to the full-time staff, half-time "cooperating teachers" join the full-time instructors as a continuation of the CTP model of intensive staff development for aspiring full-time instructors.[6]

The CUNY Central Office of Academic Affairs also supports instruction in CTI through a team of Staff Developers who have expertise in mathematics, reading, or writing instruction and who act as instructional leaders in the program. I have been the Mathematics Staff Developer, and in this role led curriculum and faculty development beginning from the formation of CTP up to and through the first year of CTI.

Program Cost

CTI students pay a $75 fee for the eighteen-week program. This covers instruction, advisement, books, materials, and tuition for students who qualify to take a credit course in Phase Two. As a result, students do not have to draw on any of their potential financial aid awards.

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[5] Adult Learning Centers on CUNY campuses provide adult basic education, pre-GED, GED, and ESL classes to adults at no charge. Taken together, the campus programs are known as the CUNY Adult Literacy/GED Program.

[6] CTI uses "cooperating teacher" to refer to someone who is training to become a full-time CTI math instructor. This job title can be confusing for teacher educators because "cooperating teacher" usually refers to veteran public school teachers who host student teachers in their classrooms.

A Growth Spurt in the Living Curriculum

Photo by Sam Seifnourian

CTI students Rosa and Daniel discuss a math problem

Transforming the 72-hour CTP math course into the 130-hour Phase One CTI course allowed the math instructor team to extended existing content to include greater complexity, and also to add a great deal of new content. This expansion was achieved while preserving the teaching, learning, and curriculum development practices described in *More Than Rules*. As was the case in previous semesters, the math team decided together which new activities would be added and how they would be sequenced. Team members helped to edit drafts of the new activities, and made additional suggestions after using the new lessons with students. Counting all teacher notes, student activities, and assessments, the Phase One curriculum document grew to nearly 650 pages.

Aside from being more intensive, the Phase One CTI math curriculum is carried out very much like the CTP math course in earlier semesters. Instructors at the sites teach the same activities using agreed-upon approaches in the same sequence. These activities include a mixture of pre-algebra and algebra topics, permitting students who have failed one or both sections of the COMPASS exams to benefit. The content is blended throughout the twelve weeks when pre-algebra concepts naturally related to algebraic ones.[7] CTI instructors find that a large majority of students who enter the program needing only to pass the algebra exam still have significant pre-algebra weaknesses, and therefore benefit from the blended content. This is quite different from "accelerated" remediation programs that do not integrate content, but instead place a doubly-intensive pre-algebra course in the first half of the semester and an algebra course in the second half of the semester.

In Phase Two, CTI math instruction departs from some Phase One practices. Students who fail any math exams after Phase One are invited to continue in a pre-algebra class, an algebra class, or in both classes depending on need. The content in Phase Two, then, is not as integrated as the Phase One course. Because the precise pattern of student needs differs between the two CTI sites, the instructors act more independently in deciding which activities to include from a common set of materials, along with how to pace and sequence them.

Students who have not passed both COMPASS math exams after Phase One and who therefore needed Phase Two math instruction are in a math class that takes into account what they studied and how they studied it in the previous twelve weeks. Students re-visit some of the more challenging concepts from Phase One, but they also study a substantial amount of new content. This approach differs from more traditional remedial programs where students who fail a remedial math class must repeat the identical course from the beginning alongside students who may be taking the course for the first, second, or third time.

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[7] Additional rationale for blending pre-algebra and algebra topics is given in *More Than Rules*, page 23.

Enrollment and Retention

It has been a challenge to recruit students for CTI. Many students who were eligible based on their placement test scores did not show much interest in the program, either because of misunderstandings or a distrust of anything that seemed to be different from what other freshmen were doing. In several cases, students were contacted about CTI after they had already registered for (but had not yet begun) a more traditional set of courses. When comparing the 25 hours of class time in CTI to their own schedule, students may have interpreted the extra class time not as more support for their learning, but as more work. For students whose financial aid award already covered the costs of their traditional remedial courses, CTI staff were not always successful in helping these students understand that joining CTI would restore that semester of financial aid eligibility. In several instances, already-registered students had a schedule that included one credit-bearing class alongside multiple remedial ones. That indicated to students that they would make at least some progress towards their degree, while joining CTI would guarantee that they would earn no credits in Phase One. The conversations we needed to have with students regarding these choices were complex, and revealed to us that most students entering community college need a great deal more college-going academic advisement while they are still in their GED or high school programs. Student recruitment continues to be a challenge, but the hope is that it will become easier as the strengths of the program are shared among students, college enrollment staff, and GED programs.

The first CTI cohort (fall 2009) included 70 students enrolled across two sites. The second CTI cohort (spring 2010) included 71 enrolled students.[8] Seventeen of the 141 total students held high school diplomas from the U.S. or another country. The remaining 124 were GED graduates. The two cohorts had remarkably similar academic profiles and outcomes, and most of the data reported in this paper will refer to the combined cohorts.

The distribution of students' need for one or both levels of math remediation as they entered CTI is shown below.

Basic Skills Math Proficiency for Entering CTI Students

	Number	% of Total
Students needing to pass Pre-Algebra and Algebra	76	53.9%
Students only needing to pass Algebra	62	44.0%
Students only needing to pass Pre-Algebra	2	1.4%
Students already proficient in both math areas[9]	1	0.7%

[8] At CUNY, the number of enrolled students is determined from a census taken in the second or third week of the semester. Before the census date, students may withdraw from a course without it ever appearing on their transcript. These students do not appear in University databases as course starters, and cannot be counted in retention data. A similar system is used to determine officially-enrolled students in CTI.
[9] One student was allowed to enroll in CTI without having any math basic skills needs. This will occur very rarely and would depend on individual circumstances.

Most community colleges report students' need for basic skills instruction in three areas—reading, writing, and mathematics. Because failing one or both math exams usually results in different numbers of required remedial courses, it is helpful to report students' need for instruction in all four basic skills exam areas—reading, writing, pre-algebra, and algebra. The distribution of all basic skills exams needed by students as they entered CTI is shown below.

Overall Basic Skills Proficiency for CTI Students[10]		
	Number	% of Total
Students needing to pass all four basic skills exams	37	26.2%
Students needing to pass three basic skills exams	65	46.1%
Students needing to pass two basic skills exams	37	26.2%
Students needing to pass one basic skills exam	2	1.4%
Students proficient in all basic skills areas	0	0.0%

We can use the above data to compute the average number of basic skills needs for entering CTI students, and compare this to a large pool of GED enrollees at CUNY. The statistics below (out of four possible exams), indicate that entering CTI students needed more remedial instruction than a large pool GED graduates who entered CUNY in 2008.

Distribution of Basic Skills Needs for CUNY GED Enrollees and CTI Students[11]	
Average number of initial basic skills needs for GED graduates entering CUNY (n = 1,230)	2.3
Average number of basic skills needs for CTI students (n = 141)	3.0

In general, students entered CTI with significant academic weaknesses. We should not rely solely on students' placement test scores and related basic skills needs to tell us this, though, because some students may not give

[10] Four students in the fall 2009 cohort entered CTI so late that they could not complete the official writing placement exam and have it scored before the beginning of classes. All four were given typical writing placement test prompts under official testing conditions and these essays were given scores by trained scorers. As was the case for105 of the other 109 students who took the official ACT writing exam, these four students did not earn passing scores. The basic skills needs indicated by these unofficial scores are included in this table.

[11] The statistic covering the large pool of GED graduates entering CUNY was generated using data provided by CUNY Collaborative Programs Research and Evaluation Unit and covers non-exempt testers who applied through the central application system in the fall 2008 semester and not those who applied through direct admission. Remedial needs for the large pool were calculated based on students' *initial* placement exam results. Some of those students certainly took advantage of summer "express" courses that allowed them to re-take placement exams and potentially reduce their basic skills needs before classes began in the fall semester. For this reason, the average number of remedial needs for the group at the time they entered fall classes was almost certainly lower than 2.3, further widening the difference between the broader GED enrollee pool and the CTI students.

their best effort on the college exams.[12] Even for GED graduates who put full effort into the COMPASS exams, though, the GED preparation these students received will rarely prepare them for the content, context, and format of the COMPASS exams.[13] GED math scores may reveal more about students' ability than COMPASS scores because there is no question that students were highly-motivated to do well on that exam, and because students' preparation was likely to be more aligned with the content of that exam. The chart below shows that CTI students had lower GED scores than a broad pool of GED graduates at CUNY. Students' GED scores did not play any role in their admission to CTI.

GED Scores for Students in CUNY and in CTI[14]		
	Mean for GED Enrollees (n = 14,252)	Mean for CTI students (n = 119)[15]
Total GED	2,547	2,503
GED Math Subject Test	496	482

Educators who work with adult, GED, or community college students with significant remedial needs might predict that it would be difficult to retain students in CTI given the large number of required classroom hours. As shown in the chart below, though, Phase One retention averaged just above 80% over two semesters.

Retention for CTI Students			
	Enrolled Students	# Completing Phase One	Retention Rate
Cohort 1	70	54	77.1%
Cohort 2	71	59	83.1%
Combined	141	113	80.1%

[12] This is a problem that has been identified at many community colleges. In a focus group study of students at five California community colleges conducted by Andrea Venezia, Kathy Reeves Bracco, and Thad Nodine, the researchers noted that "*most students did not understand that their performance on* [*placement exams*] *would determine which classes they would be able to take. Many did not realize that their performance would affect whether they would be able to get college credit for their classes right away or that it would affect how long it would take them to complete their education goals*," from "*One Shot Deal? Students' Perceptions of Assessment and Course Placement in California's Community Colleges*", WestEd, page 10; 2010.

[13] For more on the poor alignment between the GED math subject test and the COMPASS math placement exams, see *More Than Rules*, pages 8-9.

[14] The minimum passing total GED score is 2,250 and minimum passing math subject test score is 410. The CUNY figures are taken from "*College Readiness of New York City's GED Recipients*", CUNY Office of Institutional Research and Assessment, page 5 of the data tables section; 2008. The average total GED score is based on 14,252 students, the average math subject test score is based on 13,188 students, and both averages are for GED graduates entering CUNY Associate's Degree programs in the 2001-2002, 2004-2005, and 2006-2007 academic years.

[15] Scores were not available for five GED graduates and all of the high school graduates enrolled in CTI.

When students were not retained, the most common factors were health, work, and other personal issues that prevented students from meeting the attendance requirements of the program. No students were dismissed or discouraged from continuing in CTI because of weak academic skills.

Over two semesters, 113 students completed Phase One instruction and re-took basic skills exams they needed.[16] Forty-four (44) of these students became proficient in all basic skills areas after Phase One testing, leaving 69 students who continued to have needs in one or more basic skills area(s). These 69 students were the ones who could benefit from Phase Two basic skills instruction. Fifty-four (54) of the 69 students completed Phase Two classes. Phase Two retention among students who still had remedial needs, then, was 78.3%.

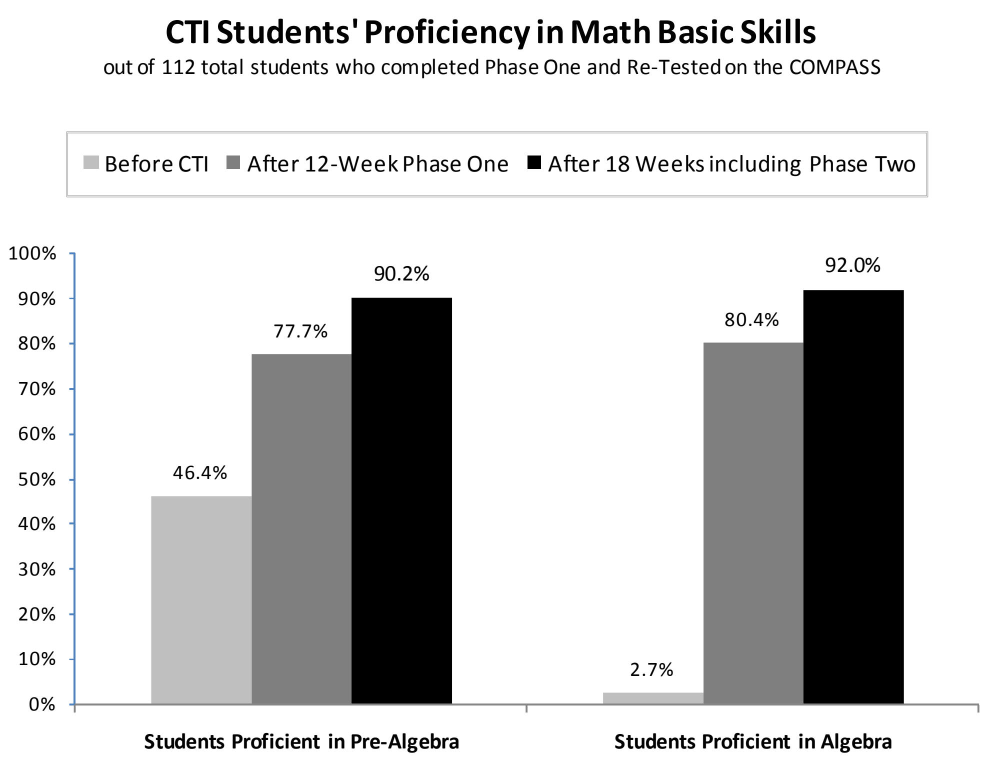
[16] Two students re-took the math exams they needed but did not re-take the writing exams they needed. Conversely, one student re-took the writing exam but not the math exams she needed.

Student Assessment

A sizeable majority of students at each site and in each semester of study passed any COMPASS math exams they needed after completing the CTI math course. This overall reduced need for math remediation is encouraging. Still, the COMPASS exams have a narrow focus on procedural knowledge and for this reason do a poor job of capturing some of the most important things that students learned and became able to do in the CTI math course. This section will include detailed COMPASS exam results but will also go beyond them to explore other types of student assessment that measure students' progress towards a broad array of math learning goals.

COMPASS Math Outcomes

The graph below shows the percentage of students who were considered to be proficient by CUNY in each math basic skills area before entering CTI, after Phase One, and after Phase Two. The data refer to the 112 students who completed Phase One and re-took any COMPASS math exams.[17] Note that most CTI students were making gains simultaneously in pre-algebra and algebra. The data supports the claim that a single course that blends pre-algebra and algebra content can help students improve their COMPASS outcomes in both areas. For more detailed information on COMPASS math scores for CTI students, see *Appendix C*.



[17] One of the 113 students who completed Phase One instruction did not re-take the COMPASS math exams she needed even though she re-took the writing exam she needed. For this reason, a base of 112 students is used here (the number who re-tested in math).

It is important to note that most of the students who did not reach basic skills math proficiency after 18 weeks were students who did not start or complete Phase Two instruction. Of the students who needed to pass the algebra exam and who took full advantage of CTI algebra instruction (12 weeks of math study if that was all they needed, or 18 weeks if that is what they needed), all but two of them passed (100 out of 102) during their CTI semester.

The changing distribution in students' need for math remediation is captured in the following table. The 112 students who completed Phase One and re-took the COMPASS math exams are included here because they had the opportunity to improve.

Basic Skills Math Proficiency for Students Over the CTI Semester			
	Before CTI	After 12 weeks of CTI	After 18 weeks of CTI
Students who needed to pass two math exams	61	13	5
Students who needed to pass one math exam	50	21	6
Students proficient in math basic skills	1	78	101
Total math exams needed by 112 CTI completers	172	47	16

Overall, 140 students enrolled in CTI needing to pass one or both math exams. After the 18-week semester, 100 of the 140 *starters* (71.4%) became fully proficient in math basic skills, and more than half of these 100 students began the semester needing to pass both pre-algebra and algebra. Data from the CUNY Office of Institutional Research and Assessment has shown that among starters in traditional CUNY remedial classes, 38% of pre-algebra students and 36% of algebra students passed those courses in 2007. Also in 2007, among students who took and passed a remedial pre-algebra course, fewer than one-third (32%) of these students who started the subsequent algebra course passed it. Direct comparisons between the CTI and broader CUNY pass rates should not be done hastily, though, because the mechanism for determining who is successful and may move on to the next course may not be the same.[18]

The CTI math course was one component in a learning community structure that also included reading instruction, writing instruction, and academic advisement. The coherence and consistency of the full program, including high expectations across both academic courses, likely had a positive impact on student persistence, in-class performance, and student outcomes in mathematics.[19] For data and description of CTI student outcomes across all basic skills areas, see *Appendix D*.

[18] In remedial math courses at CUNY colleges, students may be sent to re-take a COMPASS math exam only when they have passed the remedial course. CTI does not issue course grades for students. As long as students complete the CTI course, they re-take the COMPASS math exams.

[19] A related project known as CUNY Start provided a CTI-like math course but no reading/writing course to 100 students in the fall 2010 semester. CUNY Start includes an advisement component, but it is less-intensive than in CTI. This experiment, which continues, should provide information on the potential effectiveness of a stand-alone, CTI-like math course.

Internal CTI Assessments

The COMPASS math exams primarily assess students' procedural math knowledge, but the CTI math course has a much broader set of math learning goals. CTI math aims to improve students' abilities across all of the "strands of mathematical proficiency" as they are described in a National Research Council report.[20] The course also aims to prepare students to be successful in their first credit-based college math course, and to help students develop a variety of academic habits that will be useful in other college courses. The range of assessments used in the CTI math course aim to measure students' progress across this broad set of learning goals.

Four exams were used to assess students' math learning over the Phase One math course. These assessments were given every three weeks and included a mixture of free response items, multiple choice items, and items requiring written explanations. They were scored, returned, and discussed with students in the class as a whole, individually, and in student conferences. These exams did have quite a few items that measured students' procedural math knowledge, but an important difference between the CTI exams and the COMPASS exams is that we are certain that the CTI assessments are closely aligned with the content studied in the CTI semester.[21] When instructors noticed individual items were missed by large numbers of students, this motivated the instructors to do some re-teaching, and for persistently difficult topics, motivated the larger math team to rethink the teaching approaches for those topics in subsequent semesters.

On the very first day of the CTI math course, students completed an initial assessment. At one site, the results were used to do some tentative ability grouping. Comparing an initial assessment to later ones helped instructors to encourage students by providing clear evidence of positive changes in their ability. In a more general way, the results help the math team understand how well and in which areas students gained mastery of Phase One math content. In the chart, class averages for students who completed the initial assessment and the Phase One final exam are given for the fall 2009 and spring 2010 cohorts. These averages are appropriately compared because the assessments used

CTI Initial and Final Assessment Averages [22]		
	Initial	Final
Cohort 1 (54 testers)	19.4%	82.6%
Cohort 2 (57 testers)[23]	21.4%	82.6%

[20] The National Research Council report, Adding It Up, describes mathematical proficiency using a set of five interwoven strands: conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), strategic competence (ability to formulate, represent, and solve mathematical problems), adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). This formulation of mathematical proficiency has been endorsed by several other organizations and standards documents, including the recent Common Core State Standards Initiative. Adding It Up: Helping Children Learn Mathematics by Jeremy Kilpatrick, Jane Swafford, Bradford Findell, and the Mathematics Learning Study Committee, National Research Council, page 116; 2001.

[21] It is very difficult to know how well the content of a remedial math course is aligned with the COMPASS math exams because the publisher provides such limited information on those exams.

[22] The Phase One initial assessment and final exam use parallel, substantially equivalent items and are scored as similarly as possible using a common scoring guide. These exams differ slightly between semesters, however, because the items reflect what is taught in the course and this evolves semester-to-semester.

[23] Two of the 59 students who completed Phase One did not take the final exam. These students' initial assessment scores were excluded from the average.

parallel, substantially equivalent items and uniform scoring guides at the sites. CTI instructors used students' performance on the final assessments as a part of the planning of Phase Two instruction for students who needed it.

Other formative assessment of students' mathematical understanding occurred on a daily basis through an intense classroom focus on the development of students' oral communication of mathematical ideas. This communication was fostered through "relentless" questioning, requests for justification, exploration of students' alternate solution methods, and a general expectation that students (and not only the instructor) bear responsibility for judging and clarifying one another's reasoning. Classroom conversations were vital assessment opportunities because it was during these discussions that student understandings and misunderstandings were brought to the surface. Classroom activities were also structured in order to provide students with opportunities to ask and answer questions such as "*What does this remind us of?*", *"Is this always true?",* and "*Is this a coincidence or something more?*"—questions that gave instructors information about how well students were developing the habit of thinking and communicating like scientists.

CTI instructors aimed to simultaneously assess and help students to improve their communication skills during class discussions, a complex process. Instructors needed to react instantly to student language in the classroom, deciding when and how to search for more clarification or press for more formal language. By contrast, more traditional remedial math courses that stress lectures and or computer-based instruction may offer students very few opportunities to practice communicating mathematically.

Homework (what we call "extra practice") provided vital assessment opportunities both for students and instructors. Each extra practice set included content from the most recent lesson and from earlier learning objectives, giving timely information to students and the instructor about how students met the recent learning objectives plus information about student retention of earlier concepts and skills. Extra practice sets generally included more problems requiring conceptual reasoning and written interpretation than the exams. The majority of extra practice problems were reviewed in the classroom as soon as students arrived with them, when these discussions would be the most fruitful.[24] Instructors tracked extra practice assignments for completion rather than accuracy, and patterns (good or bad) were discussed during student conferences or at other moments when problems arose in this area.

CTI math instructors did not only suggest to students that they adopt academic habits that are important for college success—they gave students tools to improve those habits, and assessed the development of those habits. Rather than lamenting students' poor organizational skills, CTI instructors gave a binder to each student on the first day of the semester, insisted that they date and maintain all paperwork in chronological order, checked the binders multiple times over the semester, and encouraged better

[24] In many college remedial math courses, it may not be possible to discuss homework problems in the classroom in a meaningful way because there can be so little time that must be focused on each day's new content. This may force faculty members to collect and respond to student homework outside of class and return it later. Unfortunately, though, thoughtfully responding to homework is labor-intensive for faculty who may teach four or five class sections per semester, and the most useful discussions of student homework occur without any delay. Perhaps with this in mind, some college math departments are turning to computer software so that students complete homework and quizzes on-line, and where their multiple-choice responses are instantly scored. This can help to relieve faculty workload, but it may also eliminate a rich assessment opportunity because conversations among students and the instructor about the mathematics may not occur, and a record of multiple-choice responses reveals very little about student thinking. It is not a bad thing that some of these software packages include re-teaching features, but these usually do not have the sophistication to portray multiple appropriate solution methods to a problem and may not reinforce the precise method introduced by the faculty member in the classroom. Students also may not retain artifacts of their work for use at a later time. More instructional time is needed in remedial math courses to allow for detailed, collective discussions of math problems that students do outside of class.

organization where it was needed. The course also included explicit instruction on how to study for a math test. Based on a belief that studying for a math test must involve actually doing math problems, instructors supervised students as they recorded problems from prior activities that challenged them. The instructors treated the process of creating study problems as a required assignment in itself. Early in the semester, instructors supplemented student-generated study problems with additional ones, but the share of problems provided by instructors diminished over time so that student responsibility in this area would necessarily increase.

Both CTI sites held formal conferences with all students after three weeks of Phase One instruction. Regular classes were cancelled so that the three-person instructor/advisor team could meet with each student individually to discuss their progress. For the math instructor, this included a review of the student's attendance, extra practice, binder, participation in class discussions, and performance on the first exam. The conferences were excellent early opportunities to discuss students' areas of strength and concern. The conferences also gave students a chance to talk about their experience in the program. Students completed self-assessments in preparation for conferences at both sites, and at one site, students prepared formal portfolios of their early work to share with the instructor-advisor team. Beyond the third-week conferences, additional conferences with the teacher-advisor team were arranged when needed, or in some cases were repeated for all students at the end of Phase One.

In addition to the student conferences, CTI staff members at each site met every week to discuss a variety of student issues as they arose. The reading/writing instructor, math instructor, and advisor were present at all of the meetings, and the campus program manager and cooperating instructors also frequently joined in. When students were having any sort of problems, academic or non-academic, plans were made to try and improve the situation.

Overall, CTI students made substantial improvements not only in their procedural knowledge but in all of the strands of mathematical proficiency, and this was observed in the wide range of assessment practices carried out by CTI math instructors.

The goal of improving students' productive disposition deserves special mention here because so many students entered CTI with discouraging prior math learning experiences. We have made extraordinary efforts to design activities and learning environments that would boost students' interest and confidence in math learning, and the instructors witnessed very significant positive changes for students in these areas. So far, though, there have been no formal studies that would capture those changes. One of the most interesting issues here is to explore exactly how students respond (initially, and as the course unfolds) to the non-traditional pedagogy in CTI math. Rebecca Cox noted in College Fear Factor that community college students respond to non-traditional instruction with distrust or resistance, but we did not find this to be the case for most CTI students. [25] One of the reasons why students may generally be responding well to the non-traditional CTI curriculum and pedagogy is that it is highly structured—the instructors are keenly aware of exactly what they are doing, why they are doing it, and how it fits in with what comes next.

And even though CTI instructors assessed students in a variety of areas using a mixture of tools and saw widespread gains, this does not mean that students uniformly excelled in all areas. Some students

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[25] Rebecca Cox has written about student expectations of community college instructors and teaching in The College Fear Factor—How Students and Professors Misunderstand One Another, Harvard University Press, pages 95-96, and 105; 2009. According to Cox, *"Research studies in K-12 and postsecondary classrooms offer compelling evidence on the limitations of the "teaching as telling" model....yet that model continues to hold its own in the practice of and research on higher education….[it] shapes the expectations of postsecondary students and faculty…studies have documented that students associate the traditional model of teaching with faculty competence and respond to alternative methods with skepticism or disrespect."*

struggled to attend class regularly or stay focused in the classroom. Some did not develop consistently strong work habits. Some resisted our goal that they move beyond procedures in order to develop deeper understanding, reasoning, or communication skills. Instructors encouraged students to improve in all of these areas and would explain (and re-explain) the rationale for them, but some students in each CTI cohort (including students who ultimately passed the COMPASS exams) ended up with uneven gains. When looking only at COMPASS pass rates that exceeded 90%, outside observers might mistakenly believe that all of our goals were reached for 90% of the students. Clearly, it is more complicated than that.

In addition to learning goals we had for students during the CTI semester, the course aimed to prepare students to be successful in later, credit-based math courses. One of the important issues to consider here is how students will adapt when they leave the non-traditional CTI math course and likely enter a more conventional one. In the same way that many students entered CTI with expectations about what math learning looks like and then experienced something different from that, we have seen that students leave CTI carrying a new set of expectations about math learning that may conflict with their experiences in subsequent college math courses. Other issues relating to the pathway from CTI math to credit-based math courses will be described in the final section.

Because many CTI students are in their first semester of credit-based study, and because others have not yet enrolled in a math course, it is too soon to collect and share data on students' performance in credit-based math courses. We do have more general college performance data for the earlier CTP students whose performance was described in *More Than Rules*. That data is included in *Appendix E*. The early persistence rates, credit accumulation, and GPA figures are encouraging, but they refer to a very small group of students.

Teaching Practices, Standards, and Community College Reform

In The Teaching Gap, James Stigler and James Hiebert reported on the findings of a large video study of mathematics teaching in the United States, Germany, and Japan.[26] Their aim was to describe what school math teaching actually looked like in each country by observing a large random sample of videotaped lessons. They discovered a "distinctly American way of teaching," explored the cultural forces that help to keep it place, and contrasted it with standards documents from the National Council of Teachers of Mathematics, and also with common teaching practices in the other two countries.

The video study examined mathematics teaching in 8th grade classrooms, but the patterns Stigler and Hiebert discovered can also be found in community college remedial math teaching. To support this claim, I draw on a recent observational study of remedial math teaching across 13 colleges by Norton Grubb. In their characterization of math teaching as a cultural activity, Stigler and Hiebert help to explain why teaching practices in the U.S. are slow or resistant to change. I will argue that teaching practices in community college remedial math classrooms are also constrained by cultural and systemic forces. Finally, I will characterize the most common remedial math reform efforts in community colleges and contrast them to the innovative teaching and learning approaches in the CTI math course.

"Learning Terms and Practicing Procedures"

The research team working with Stigler and Hiebert felt that "learning terms and practicing procedures" was a good overarching motto for the teaching they observed in U.S. math classrooms. The U.S. lessons "seemed to place greater emphasis on definitions of terms and less emphasis on the underlying rationale" than lessons in the other two countries.[27] It was unusual to see U.S. instructors developing new ideas using explanations or an example; it was much more common that teachers simply stated the information or formula.[28] The researchers found that U.S. teachers were less likely to connect mathematical ideas within a lesson in meaningful ways.[29] Compared with teaching in the other countries, the researchers found the U.S. lessons were the least likely to help students "understand important mathematics", both in terms of "the level of challenge and how the content was developed."[30] New ideas can be developed in ways that are "student-controlled", and this was much less common in the U.S. tasks than in German and Japanese tasks.[31] The researchers also classified "seatwork" tasks into ones where students were practicing routine procedures, ones where students applied concepts or procedures in new situations, and

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[26] The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom, by James W. Stigler and James Hiebert, The Free Press; 1999. The TIMMS Video Study was one component of the Third International Mathematics and Science Study (TIMMS) which also included teacher questionnaires for teachers in the video study countries and math tests for 4th, 8th, and 12th grade students across 41 countries.

[27] *Ibid*, page 59.

[28] *Ibid*, page 60. U.S. instructors stated the information 78% of the time, and developed new ideas 22% of the time.

[29] *Ibid*, pages 62-64.

[30] *Ibid*, page 65.

[31] *Ibid*, pages 66-67. Student-controlled tasks appeared 9% of the time in U.S. classrooms, 19% of the time in German classrooms, and 40% of the time in Japanese classrooms. An example given of a "student-controlled" method involved the sum of the interior angles of a polygon. Students could be asked "to measure the sums of interior angles of various polygons using a protractor, and then try to find some patterns that would help them compute the sums more quickly. Students could have been given responsibility to work out various solution methods—including, perhaps, a general formula." This is contrasted with a teacher-controlled method, where the teacher would show students the formula [$sum = 180(number\ of\ \ sides - 2)$], and then ask students to calculate the sums of various polygons using that formula.

ones where students invented something new or analyzed a situation in new ways. In U.S. lessons, students spent almost all seatwork time practicing routine procedures.[32]

Stigler and Hiebert saw some variation in the U.S. teaching they observed, but they were shocked at how limited it was. The variation they did see tended to appear in "the form of the activities, not the substance." Even when U.S. teachers organized lessons to involve students more actively in their learning, "the mathematics [was] simple compared with that encountered by their German and Japanese peers, and the work and discussion are mostly about memorizing definitions for terms and following rules and procedures."[33]

The most striking contrast across the three countries was the difference between Japanese and U.S. math teaching. Stigler and Hiebert noted that Japanese classrooms allow us to see "what it can look like to teach mathematics in a deeper way, teaching for conceptual understanding. Students in Japanese classrooms spend as much time solving challenging problems and discussing mathematical concepts as they do practicing skills."[34] The motto used to describe an overall Japanese approach was "structured problem-solving". Topics were rarely stated by Japanese teachers but were much more likely to be developed with examples or other illustrations. This gave Japanese students "richer opportunities [than U.S. students] to learn the meanings behind the formulas and procedures they [were] acquiring."[35] Japanese lessons were rated by the researchers as more coherent and contained more clear connections between ideas than the U.S. lessons. Japanese students had a very different role in the classroom than typical U.S. students. This especially appeared in the greater responsibility Japanese students had for developing new ideas, by "first struggling to solve mathematics problems, then participating in discussions about how to solve them, and then hearing about the pros and cons of different methods and the relationships between them."[36]

"Remedial Pedagogy"

Over the 2009-10 academic year, Norton Grubb led a research team as it conducted an observational study of remedial instruction at 13 California community colleges.[37] Grubb's team observed a consistent set of teaching practices that he names "remedial pedagogy". Grubb's descriptions of "remedial pedagogy" are not as detailed as those in The Teaching Gap, but we can see several parallels. In this excerpt, Grubb summarizes one math lesson as an example of what the researchers observed "over and over":

> *"The class is a…presentation of a series of small sub-skills, presented without any justification for why such skills might be useful in other contexts…and a single method—presentation and practice—is used for the entire class. When students ask questions about the procedures, the instructor simply repeats his previous explanation rather than providing an alternative. The instructor periodically asks a formulaic question about understanding, but when students make mistakes, or are obviously*

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[32] *Ibid*, page 71. The U.S. students spent 95.8% of their seatwork time practicing routine procedures.
[33] *Ibid*, page 53.
[34] *Ibid*, page 11.
[35] *Ibid*, page 60.
[36] The Teaching Gap, page 91.
[37] The University of California-Berkeley and Research and Planning Group (UCB-RP) Study of Basic Skills in California Community Colleges is described in *"The Quandries of Basic Skills in Community Colleges: Views from the Classroom*", by Norton Grubb as a working paper for the National Center for Postsecondary Research Developmental Education Conference; September 2010. In some instances, colleges participating in the UCB-RP study did not agree to random selection of instructors, and instead directed the researchers to observe specific instructors. The researchers also had a difficult time incorporating adjunct instructors into the study. Both of these conditions could have introduced some bias into the study.

> *guessing, he provides the right answer—rather than engaging in any diagnosis of why students have arrived at the wrong answer."*[38]

Other components of "remedial pedagogy" highlighted by Grubb include an emphasis on the right answer rather than attending to students' conceptual understanding, and an inattention to alternative solution methods.[39]

In order to locate "remedial pedagogy" inside a larger framework of teaching possibilities, Grubb characterizes teaching across two dimensions. First, he describes teaching along a "behaviorist/constructivist" continuum:

> *"On the one hand are the pedagogical approaches called constructivist, student-centered, "progressive," conceptual, "active," "teaching for meaning," or innovative, while others are called behaviorist, teacher-centered, traditional, conventional, information transfer, or passive."* [40]

To simplify the discussion, Grubb summarizes the former approaches as "constructivist", the latter as "behaviorist", and "balanced" to describe a blend of the two.

To avoid grossly oversimplifying things, Grubb recognizes that there are many other aspects to teaching than degrees of "behaviorism" and "constructivism". He therefore melds a number of facilitation and other teaching practices into a second dimension of teaching—"quality". His examples of teaching quality include "content mastery; warm and supportive relationships with students; explicitness about the purposes of instruction; clarity in presentation; care in providing the prerequisites for understanding before developing new material; developing checks for student understanding; and using student errors to diagnose how students are thinking (sometimes incorrectly) about a topic." [41] Admitting that his depiction cannot fully capture the many dimensions of teaching practice, he nevertheless reminds us that instruction can be low- or high-quality in ways that are independent of their behaviorist/constructivist orientation.[42]

Drawing on research on how students learn, "well-specified statistical models" and also on "the consensus of instructors about what works", Grubb provides several arguments to support the claim that "balanced/constructivist" approaches to pedagogy are preferred to "behaviorist" ones for remedial students.[43] Taking both of his dimensions of teaching into account, then, Grubb argues that the ideal type of community college remedial math instruction may be both "balanced" and "high-quality". Most of the

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[38] *Ibid*, page 11.
[39] *Ibid*, page 12.
[40] *Ibid*, page 3. For me, constructivist pedagogical approaches are teaching approaches that stem from constructivist theories of learning. One of these is "*radical constructivism…the philosophy that knowledge cannot be provided in some final form from…teacher to student but must be actively assembled in the mind by each learner in his or her own way. The responsibility for expanding what one knows, or for constructing new knowledge, rests primarily on the learner and his or her efforts to achieve understanding."* Another theory is *"social constructivism [which] maintains that students can better build their knowledge when it is embedded in a social context. Thus, the interaction between teacher and students is enhanced when it involves a broader community of learners--that is, students working together. Students help one another create richer meanings for new mathematical content."* These definitions are drawn from the article, *"Constructivist Mathematics and Unicorns"*, by Lee Stiff in the *NCTM News Bulletin,* National Council of Teachers of Mathematics; July/August 2001.
[41] *Ibid*, pages 7-8.
[42] Grubb also points out that some aspects of teaching quality are not independent but are quite specific to the behaviorist or constructivist tilt of a lesson.
[43] *Ibid*, pages 4-7.

pedagogy Grubb's team actually observed was behaviorist and low-quality, though, suggesting to him that there is much room for improvement.[44]

Math Teaching as a Cultural Activity

Stigler and Hiebert were "amazed" at how little variation they saw in U.S. math teaching in the video study. To help explain this, and also why traditional teaching methods are so resistant to change, they define teaching as a "cultural activity". For them,

> *"…[teaching is] learned implicitly, through observation and participation, and not by deliberate study…This might be surprising because teaching is rarely thought of in this way….some people think teaching is an innate skill, something you are born with. Others think that teachers learn to teach by enrolling in college teacher-training programs. We believe that neither are the best description. Teaching, like other cultural activities, is learned through informal participation over long periods of time. It is something one learns to do more by growing up in a culture than by studying it formally."*[45]

Culturally-developed beliefs about teaching and learning, according to Stigler and Hiebert, include beliefs about mathematics as a subject, how students learn, and the appropriate roles of the teacher and students in the classroom. Most U.S. teachers in their study "behave[d] as if mathematics is a subject whose use for students, in the end, is as a set of procedures for solving problems."[46] With this view of mathematics, "it would be understandable to believe that mathematics is learned best by mastering the material incrementally, piece by piece."[47] The role of the U.S. teacher, then, is to shape "the task into pieces that are manageable for most students, providing all the information needed to complete the task and assigning plenty of practice."[48]

Stigler and Hiebert argue that it is difficult to change these practices precisely because teaching is a culturally-driven activity.

> *"The more widely shared a belief is, the less likely it is to be questioned—or even noticed. This tends to naturalize the most common aspects of teaching to the point that teachers fail to see alternatives to what they are doing."*[49]

Changing teaching is also made more difficult because teachers are only one part of a complicated educational system that tends to reinforce traditional approaches to instruction. Stigler and Hiebert point to the physical classroom setting, textbooks, district/state objectives, students, and the organization of the school day as factors that can reinforce traditional teaching methods, and "changing any one of these individual features is unlikely to have the intended effect."[50]

> *"Teachers asked to change features of their teaching often modify the features to fit within their pre-existing system instead of changing the system itself. The system assimilates individual changes and*

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[44] *Ibid*, page 8. Grubb's team did encounter some math instructors who were experimenting with "balanced" pedagogical approaches. Some of these were drawing on math curricula connected to National Council of Teachers of Mathematics that were originally created for K-12 contexts. Still, this was rare, and math instructors were the least likely when compared to other basic skills instructors (reading, writing, and ESOL) to deviate from "remedial pedagogy".
[45] The Teaching Gap, page 86.
[46] *Ibid*, page 89.
[47] *Ibid*, page 90.
[48] *Ibid*, page 92.
[49] *Ibid*, page 100.
[50] *Ibid*, page 99.

> *swallows them up. Thus, although surface features appear to change, the fundamental nature of the instruction does not."*[51]

Pressuring instructors to adopt individual reforms that do not recognize that teaching is a deeply-held cultural activity can have several possible negative consequences.

> *"Teachers can grow weary. They are asked over and over to change the way they do x, y, or z. Even when they try to accommodate the reformers and adopt a new feature or two, nothing much happens. They do not notice much improvement in students' learning…Always changing, and yet staying the same, is a discouraging state of affairs. It can lead to a defeatist kind of cynicism."*[52]

Stigler and Hiebert point out through others' research and their own classroom observations that superficial adoption of reforms can actually lead to worse teaching than traditional instruction left alone.[53]

The most significant recent test of the durability of the "American" way of teaching might be the K-12 reform efforts of the National Council of Teachers of Mathematics (NCTM).[54] The first NCTM standards documents were widely disseminated for years before the video study. These standards challenged traditional teaching in fundamental ways, including the underlying assumptions about how students learn and the appropriate classroom roles for teachers and students. Almost all (95%) of the U.S. teachers in the video study said they were "somewhat aware" or "very aware" of reforms advocated by NCTM. Moreover, 70% of the teachers indicated that they implemented the reforms in their classrooms and that evidence of this would be observable in the videotapes. When Stigler and Hiebert's team viewed the classroom videos, though, they "found little evidence of reform, at least as intended by those who had proposed the reforms."[55] Regardless of one's view of the quality of the NCTM standards, the video study provided a national sample of classroom teaching showing that the standards may not have had widespread impact on teaching practice. Stigler and Hiebert noted that the Japanese lessons they observed did a better job of displaying the kinds of thinking, problem solving, and discourse emphasized in the U.S. reform documents.[56]

Culture and Remedial Math Reform in Community Colleges

The "remedial pedagogy" that was consistently observed by Grubb's team across math classrooms at 13 colleges can also be seen as a reflection of cultural and systemic forces—forces that may be even more resistant to change than in K-12 settings.

Much like K-12 math teachers, community college remedial math instructors also attended school for 16+ years where their conception of math teaching and learning was shaped by their own observation and participation. Something that may distinguish college remedial instructors from K-12 math teachers is that the college instructors were probably among the most successful math learners in the schools and colleges they attended. Traditional math teaching worked very well for them. College instructors also have less pre-service training than K-12 teachers, and so have fewer (or no) opportunities to gain early exposure to alternate instructional methods.

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[51] *Ibid*, page 98.
[52] *Ibid*, page 100.
[53] *Ibid*, pages 99-100 and 106-108.
[54] NCTM's Professional Standards for Teaching Mathematics (1991) articulated teaching practices that would support the Curriculum and Evaluation Standards for School Mathematics (1989). These documents, along with one focusing on assessment (1995) were combined and revised in the Principles and Standards for School Mathematics (2000). This most recent document is available at http://www.nctm.org/standards.
[55] The Teaching Gap, pages 105-106.
[56] *Ibid*, page 106.

And even though cultural forces and the absence of pre-service training may incline college faculty towards "behaviorist" approaches, this does not mean that they are not working hard to try and help their students be successful. The reality is that many faculty members are doing as good a job as they can and are dismayed by their students' lack of success. Their individual efforts to improve the situation may not succeed, though, because isolated faculty members are not able to address the larger system that makes remedial math teaching and learning so difficult.

One factor limiting faculty members' ability to improve their own instruction is inadequate instructional time. Remedial math syllabi are choked with content, and remedial courses generally meet 60 or fewer hours in a semester. Within these constraints, the only apparent way an instructor can move through the syllabus is to teach in an extremely "behaviorist" manner. This approach is not really as efficient as it seems, though; rushed lectures and demonstrations of procedures are exactly what did not work for many remedial students in earlier grades. Department-wide assessments that are tied to content-packed syllabi only intensify the pressure to move quickly and make "behaviorist" approaches even more likely.

Community colleges also generally provide few opportunities for ongoing professional development that would help instructors move towards more "balanced/constructivist", and/or (taking Grubb's shorthand) "high-quality" teaching methods. Grubb's team found this to be disappointingly true in the California community colleges they visited.

> *"[Professional development] in California colleges is particularly poor; colleges offer "flex days" where faculty attend workshops of their own choosing, with most subjects unrelated to instruction. Workshops include such topics as CPR, creating retirement accounts, tips for vacations, and the initial day of the college where the president introduces new faculty and programs. Adjunct faculty rarely attend, because they are paid only for teaching…community colleges in our sample have no systematic ways of improving the quality of instruction, either in basic skills or in any other courses. Elsewhere, colleges have developed ongoing forms of professional development…but these appear to be rare, and they were certainly absent in the 13 colleges we observed."*[57]

Especially over the past five years, and in response to consistently poor outcomes for students who place into remedial math courses, there have been increasing calls for institution-wide (and not just individual) reforms of remedial mathematics in community colleges. Rather than addressing the instructional methods that dominate remedial math classrooms and the cultural and systemic factors that keep those methods in place, though, the most popular reform efforts actually sidestep the critical issue of teaching practice. Consider the following common approaches to reform:

Revising syllabi—Initiatives to revise remedial math syllabi are generally limited to very modest tinkering with the ordering of topics. It is rare that learning goals are reconsidered in fundamental ways or that the number of topics is trimmed so that instructional methods can be improved, deeper student understanding can be emphasized, and the need for re-teaching can be reduced.

Selecting a textbook or curriculum—The process of reviewing textbooks or curricula can be an opportunity to explore contrasting instructional approaches. Rare as it is that community college programs adopt a textbook or curriculum that emphasizes "balanced/constructivist" approaches, even this step would not guarantee any changes in actual teaching practice. A textbook or curriculum may only be used only for its problem sets and not as a rich guide for developing mathematical ideas, and this may be more likely with more innovative materials.[58] As Stigler and Hiebert put it, "even textbooks can get swamped by the system."[59]

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[57] "*Quandries*", page 21.

[58] For more on the growing literature that compares "intended" to "enacted" curricula, see Mathematics Teachers at Work: Connecting Curriculum Materials and Mathematics Instruction, edited by Remillard, Herbel-Eisenmann, and Llyod, Routledge; 2009. My own experience over several years creating curricula for use with GED and high school

Expanding use of tutors—Community colleges spend considerable resources hiring and managing tutors for use in math labs or other tutoring configurations. Unfortunately, tutors rarely receive any meaningful pedagogical training, their content knowledge can be uneven, and when they are separated from faculty, the assistance they provide to students may not align well with what faculty members are doing in the classroom.

Expanding computer-based instruction—Many colleges are considering or have already begun using computer labs to supplement or replace human instruction. These software packages generally emphasize procedural math knowledge and, according to Grubb, replicate "remedial pedagogy".[60]

Creating specialized placements—Some colleges are experimenting with new ways of grouping remedial math students based on their placement test scores or their interest in a particular course of study. "Modularized" programs invite students who seem to show proficiency in certain content topics to only do a portion of a remedial course rather than the entire, semester-long one. Learning communities or linked courses may be arranged so that faculty can become more familiar with a single group of students and in some cases integrate content between a remedial and credit-based course (though this is less common with mathematics than remedial reading, writing, or ESOL). Special "repeater" sections may also be created for students who did not pass a remedial math course on a previous attempt.

Adopting "express" or "accelerated" courses—In order to try to move students with significant remedial math needs more quickly into credit courses, colleges are also experimenting with the intensity and duration of instruction. "Express" courses may provide students with an abridged option, often in the summer session. "Accelerated" courses may provide the same or fewer hours of instruction as a typical semester-long course, but compress it into less calendar time by boosting weekly instruction.

These popular initiatives all have something in common—they almost never include a substantial rethinking of teaching methods. Instead, the reforms amount to a mere "reshuffling" of traditional teaching practices. The reforms give the appearance of change in that students may sit in a computer lab instead of a classroom, with a tutor a bit more often, or in longer class sessions over fewer weeks, but the actual mathematical content they encounter and the reasoning demanded of them are not different.

In the case of community college remedial math education, then, there is quite a lot of recent interest in reform but there is no general recognition that teaching methods can or should improve. Improved pedagogy is not even a goal of most reform efforts. Educational researchers tend to follow this lead and produce evaluation studies of pedagogy-free reforms. The scarcity of studies (qualitative or quantitative) showing that any of the above reforms are making a substantial impact on student learning should signal to us that one or more fundamental issues are going unaddressed.

One organization that has made a clear case that instruction should be improved is the American Mathematical Association of Two-Year Colleges (AMATYC), one of the professional organizations for mathematics faculty at community colleges. AMATYC produced standards documents that were clearly influenced by the earlier NCTM standards. The first of these, *Crossroads in Mathematics—Standards for Introductory Mathematics Before Calculus*, was endorsed by a host of other organizations including the Mathematical Association of America and NCTM.[61] The pedagogical standards contained in *Crossroads* pointed to the importance of creating interactive and collaborative learning environments, of making

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math students, and leading professional development activities with instructors, has shown me that behaviorist-inclined instructors, without intensive professional development, tend to revise "balanced/constructivist" activities so that they conform with their typical practices.

[59] The Teaching Gap, page 98.

[60] *Quandries*, page 10.

[61] "*Crossroads in Mathematics—Standards for Introductory College Mathematics Before Calculus*", prepared by the Writing Team and Task Force of the Standards for Introductory College Mathematics Project, Don Cohen, Editor, American Mathematical Association of Two-Year Colleges; September 1995.

explicit connections to other disciplines, of valuing multiple approaches to problems, and of developing students' ability to think independently (by making conjectures and generalizations, among other means).[62] The standards also took a clear position in support of constructivist learning theories:

> *"The standards for pedagogy….are compatible with the constructivist point of view. They recommend the use of instructional strategies that provide for student activity and student-constructed knowledge."*[63]

A second AMATYC document titled *Beyond Crossroads—Implementing Mathematics Standards in the First Two Years of College* was created as an update to the earlier document, and as a guide for achieving the standards.[64] *Beyond Crossroads* focused on the need for faculty to draw on a variety of instructional approaches (including "student-centered" and "teacher-centered" approaches) to serve a diverse group of learners. The document emphasized the importance of "active learning", and pointed to collaborative/cooperative, discovery, "question-posing", and writing activities as good ways to spur this.[65]

In their substance, both AMATYC documents recommended a move towards more "balanced/constructivist" instructional approaches, but these standards appear not to have impacted typical remedial instruction in a significant way. Of course a similar disconnect existed when Stigler and Hiebert compared the NCTM standards to teaching in the video study.

The Statway initiative is another example of a significant effort that is underway to transform remedial math teaching in a "balanced/constructivist" direction.[66] Statway actually imagines a new remedial mathematics pathway for students entering colleges in the social sciences, arts, humanities, business, applied technologies, health sciences, and other so-called "non-STEM" fields.[67] Rather than accepting that these students should spend time (and often languish) in remedial math courses that are focused on algebra topics that will prepare them for pre-calculus and calculus, Statway advocates argue that a year-long rigorous course in statistics that blends remediation and credit-based study will be more useful for these students given their academic and career goals. Statway not only calls for changes in math content for these students, but also for pedagogy that diverges from traditional practices. In their *Instructional Design Principles*, Statway organizers have stated their desire to create a course where "teachers and students [will] attend specifically to concepts" and not only to procedures, and where students will be "discussing meaning underlying procedures, comparing and contrasting solution strategies, [and] considering how problems build on each other or are special (or general) cases of each other."[68] The *Principles* also point to the importance of "struggle" in the learning process. In their conception, "struggle" refers to moments when "students expend effort to make sense of math, to figure something out that is not immediately apparent," and when these experiences are carefully constructed, it can lead students to "work more actively…to make sense of the situation, which in turn leads to interpretations more connected to what they already know."[69] Statway organizers have constructed a substantial network

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[62] *Ibid*, pages 15-17.
[63] *Ibid*, page 15.
[64] "*Beyond Crossroads—Implementing Mathematics Standards in the First Two Years of College*", prepared by the Beyond Crossroads Writing Team, American Mathematical Association of Two-Year Colleges; November 2006.
[65] *Ibid*, Chapter 7.
[66] Statway is supported by the Carnegie Foundation for the Advancement of Teaching, and involves faculty members at 19 colleges in five states.
[67] STEM is an abbreviation for "science, technology, engineering, and mathematics" fields of study.
[68] The *Statway Instructional Design Principles* were outlined in a slideshow presented by Bill Saunders at the Statway Summer Institute; July 2010.
[69] *Ibid*, slides 12 and 13. The extent to which students are to "struggle" in math classes was also a powerful theme raised by Stigler and Hiebert when examining cultural assumptions about how students learn mathematics and the appropriate roles of teachers and students in U.S. and Japanese classrooms. Because mathematics in the U.S. is generally viewed as a series of procedures, and math is often thought to be learned best by mastering skills

for collaboration, reflection, and support as faculty teach pilot lessons for the new course, a recognition of the challenge of promoting non-traditional pedagogy across 19 partnering institutions. It will be interesting to see how the pedagogical values outlined in the *Principles* compare to the teaching in actual Statway classrooms as the project moves past its pilot phase.

Math Teaching and Learning in CTI

In my view, the promising outcomes for CTI math students have been accomplished not by "reshuffling" traditional remedial math teaching practices but by diverging dramatically from them. The CTI math course rests on a set of beliefs held by a team of instructors regarding the nature of mathematics, how students learn, and the appropriate roles of teachers and students. The paragraphs that follow will review these beliefs and the teaching practices that stem from them.[70] This description should reveal that CTI math pedagogy is closer to what Stigler and Hiebert observed in Japanese classrooms than in U.S. classrooms, is closer to what is depicted in the AMATYC standards than what was observed by Grubb's team in California remedial classrooms, and is closer to what Statway organizers aspire to than reform efforts that remain silent on teaching practice.

The teaching materials and instructor practices in CTI math stem from the view that mathematics is a subject that is not limited to a set of procedures, but is one that involves relationships between mathematical concepts, procedures, and conventions.

The CTI math course has been designed in accordance with a belief in the importance of student-constructed knowledge. This includes a belief that students construct knowledge best when new ideas are connected to previous understandings (and misunderstandings), and when students are socially active in conjecturing, investigating, discovering, justifying, critiquing, and reasoning with each other and the instructor about new ideas.

In light of the above beliefs about the nature of mathematics and how students learn, the activities and discussions in CTI math are organized so that students share responsibility with the instructor for developing new mathematical ideas. Students are often asked to grapple with non-routine problems and compare their different solution methods. The mathematics that students do and think about is the centerpiece of the classroom through an intense focus on "student talk", and student descriptions of their mathematical thinking (in formal groups, in informal pairs, and in all-class discussions) are as important as the answers they arrive at for specific problems.[71] Realistic contexts and number relationships are frequently used so that students connect their prior understandings to more formal and complex representations.[72] Overall, students are involved in a rich set of tasks in the classroom, including

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incrementally, "*practice should be relatively error-free, with high levels of success at each point. Confusion and frustration…should be minimized; they are signs that earlier material was not mastered.*" Based on their observations of Japanese classrooms, though, "*One can infer that Japanese teachers believe students learn best by first struggling to solve mathematics problems, then participating in discussions about how to solve them, and then hearing about the pros and cons of different methods and the relationships between them. Frustration and confusion are taken to be a natural part of the process, because each person must struggle with a situation or problem first in order to make sense of the information he or she hears later. Constructing connections between methods and problems is thought to require time to explore and invent, to make mistakes, to reflect, and to receive the needed information at an appropriate time.*" The Teaching Gap, pages 90-91.

[70] Additional description of the teaching and learning practices in CTI can be found in *More Than Rules,* pages 27-32.

[71] See *More Than Rules*, pages 29-30 for more on the importance and facilitation of "student talk".

[72] See *More Than Rules*, page 29 for more elaboration of this idea, and *Appendices H* and *I* in that paper for detailed examples from the curriculum. "Contextualization" is often pointed to as an innovative pedagogical approach, but in practice may not differ much from strictly "behaviorist" teaching. One type of "contextualization" occurs when an

investigating relationships and patterns that are put before them, generalizing relationships based on their investigations, applying concepts in new situations, and practicing procedures in order to build accuracy and fluency.[73]

The fact that students share responsibility for developing their understanding of mathematical ideas does not mean that CTI instructors are passive in the classroom. There are almost no instances where CTI math instructors need to lecture or state a mathematical rule or relationship to students, but they have the very challenging task of orchestrating students' participation using carefully-designed examples and discussions. The only instances where a CTI instructor must really "give" students knowledge is when they need to share conventional notation or vocabulary that students cannot discover and have not been exposed to in earlier courses.[74] CTI instructors also aim to create a supportive environment where errors and struggle are treated as natural and important parts of the learning process.

In my view, the most interesting outcome after two semesters of the CTI math course is that "balanced/constructivist" teaching approaches have helped students to make significant gains in all strands of mathematical proficiency. The important point here is that students do not need to sacrifice improvements in their procedural skills in order to improve their proficiency in a more comprehensive way.

After three years of work developing the CTP and CTI math courses, I am convinced that poor outcomes for remedial math students are not inevitable. My experiences have led me to believe that more "balanced/constructivist" pedagogical approaches can play an important role in helping students—even students who enter college with very weak skills, understanding, and confidence—to learn a lot of mathematics. To achieve these gains, however, a number of institutional and other conditions need to be in place so that high-quality "balanced/constructivist" pedagogy can emerge.

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instructor adds a few application problems to the end of what is a teacher-centered, procedurally-focused demonstration. In these cases, the application problems can be quite mundane, require that students merely repeat the procedures they just practiced, and fail to meaningfully extend student thinking. One positive outcome that could stem from this "behaviorist-contextualization" is that students may develop a sense that the mathematics being studied is actually useful for something. In general, though, realistic contexts in the CTI math course can be used in very different ways; contexts are used as a part of developing (and not just practicing) new ideas. As is shown in *Appendices H* and *I* of *More Than Rules*, students solve realistic problems with the knowledge they already have, and these solutions are the raw material that instructors draw on to guide students towards more formal and challenging representations and concepts.

[73] See *More Than Rules*, *Appendix G* for an example of how an exploration of function inputs and outputs is a part of a student-centered, conceptual approach to introducing slope. See *Appendix F* for an example of how procedural knowledge of an exponent rule is constructed through student generalizations after working with multiple examples and not in a more teacher-centered way as a piece of knowledge that is stated by the instructor.

[74] See *More Than Rules*, *Appendix G* for an example of how students can be guided to do the work of uncovering mathematical relationships, while the instructor collects the observations and attaches the conventional vocabulary to them (in this case, "rate of change").

More Than Reshuffling

If a group of faculty at a community college decide that they want to experiment with or commit to more "balanced/constructivist" pedagogy in their remedial mathematics program, a range of practices need to be reconsidered to make this possible. This includes embracing new approaches to learning goals, remedial math content, instructional intensity, student assessment, and most importantly, faculty development.

Extending learning goals

Learning goals for remedial students should include more than lists of mathematical procedures. The strands of mathematical proficiency outlined by the National Research Council, the *Standards for Intellectual Development* in *Crossroads*, or some combination of the two would be a strong foundation for determining a comprehensive set of learning goals in any community college remedial mathematics course. Of course, adopting an ambitious set of learning goals is the easy part; many remedial math syllabi already include impressive-sounding lists of learning goals. The challenge lies in creating tasks, discussions, assessments, and classroom environments that actually help students reach those goals.

Reorganizing content

The choices faculty make in determining remedial math content can create (or limit) opportunities to improve pedagogy. Firstly, remedial math programs ought to reconsider the typical separation of pre-algebra and algebra content. The number of topics in remedial syllabi should also be scaled back so that the content can be examined in more depth. Finally, changes should be made within remedial courses and in remedial math pathways so that students study content that is coherent in its organization and is aligned with their academic and career goals.

Most students who manage to pass the COMPASS pre-algebra placement exam but not the algebra exam still have deep number weaknesses which haunt them later. Students who need one or both levels of remedial math take the same CTI course, and this gives them opportunities to improve their number abilities at the same time that these number topics are used to illuminate algebraic ones.

Adopting a CTI-like Phase One/Phase Two structure would allow a program to blend pre-algebra and algebra content for all students and still provide sensible differentiation. Some students entered CTI only needing to pass the algebra exam but required both Phase One and Phase Two (18 weeks) to do so. Others needed both pre-algebra and algebra but were able to pass both exams after Phase One (12 weeks). Any "Phase Two" -type structure should take account of, strengthen, and extend what students do in the core instructional period—not ignore and repeat it.

Rather than grouping more remedial mathematics students into blended courses, several colleges are actually moving in the opposite direction by adopting "modularized" programs. These programs attempt to dissect a remedial course into smaller "modules" and use computerized tools to place students only into the modules they appear to need.

When modules are taught by human instructors, it is difficult to establish classroom norms around mathematical language, listening, questioning, and working together (which have been so important in CTP and CTI) because new students may enter or exit the room after just a few weeks of study. Allowing some students to skip a module also does not recognize the fact that good instructors will "scaffold" learning over a semester, teaching early topics in particular ways so that the techniques and understanding that students acquire early on will pave the way for them to master more challenging topics. Students who clicked their way out of a module (whether they have deep understanding of the topics or not) will not have the benefit of those earlier experiences.

Modularized programs also frequently rely on computer-based instruction rather than human instructors. These software tools generally use "teacher"-centered approaches in which mathematical procedures are demonstrated by the software and are practiced and (hopefully) mastered by the student. Most of the software tools do not give students sufficient opportunities to communicate their reasoning, ask and answer questions with other students, explore alternative solution strategies and representations, or to participate actively in the development of new ideas through inductive reasoning or other more student-centered approaches.

The recent popularity of modularized programs rests on the idea that if we can "individualize" instruction, we can improve students' persistence by not wasting time teaching them what they already know. Modularized programs put enormous faith in computerized diagnostic tools to determine what students know, but the problem here is that students' mathematical understandings and misunderstandings are terribly complex and cannot be parsed in this way. This is especially true when we think about students' mathematical proficiency in a broad sense, and not so narrowly as an incremental set of procedural skills.

Modularized programs also can be seen as a response to the challenge of student heterogeneity in the remedial mathematics classroom. Rather than seeing heterogeneity as a problem, though, students' different experiences and ways of viewing mathematical relationships can be an asset in a classroom when these differences are celebrated and mined for discussion. CTI math instructors expect that students will provide a range of appropriate and different solution methods or representations to problems precisely because of their entering differences.[75] The important task, then, is for the students to examine the different methods or representations to determine how they are connected, if they are equivalent, the advantages or disadvantages each may have, or if there are errors that can be found. There is a similarity here to the way that Stigler and Hiebert described Japanese teachers' approach to differences among students:

> *"For the Japanese teacher, the differences within a group are beneficial because they allow a teacher to plan a lesson more completely. Japanese teachers plan lessons by using the information that they and other teachers have previously recorded about students' likely responses to particular problems and questions…They can then plan the nature of the discussion that is likely to occur. The range of responses also provides the vehicle teachers use to meet the needs of different students."*[76]

Of course there are some cases where students may be placed in the CTI math course with exceptionally strong or weak ability, and this can provide a real challenge to the instructor. These situations are a result of problems in the remedial placement system, though, and I will address these more directly below.

Aside from blending content, another important content-based reform that is necessary in any move towards more "balanced/constructivist" pedagogy is a reduction in the quantity of topics in remedial math syllabi. Even with an increase in instructional intensity, the extensive list of content topics in most

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[75] An example of this is the first activity involving exponents in the CTI math curriculum. Rather than having the instructor lecture on exponent notation and its meaning, the students are organized into groups and work on a challenging functions problem involving the number of cubes that will be needed to continue a pattern. The student-created functions that result often include all the following: $y = (x)(x)(x)$, $y = (x^2)(x)$, and $y = x^3$. These different, equivalent function equations flow from the different ways that the groups solve the problem. Once students agree that the function equations are equivalent, the students (and not the instructor) are the ones who have discussed and agreed on the meaning of exponent notation.

[76] <u>The Teaching Gap</u>, pages 94-95.

remedial math syllabi make it almost impossible to do more than lecture on procedures. Removing topics from a syllabus might be criticized as a call to lower mathematical standards, but it need not be viewed that way. Prioritizing fewer, vital topics will allow students to be more involved in their learning process, to do substantial, mathematical thinking rather than endless drills on procedures they do not understand, and will reduce the need to re-teach topics in later courses. Trimming content that does not connect well to major course objectives actually could contribute to more coherent courses.

A larger issue of coherence exists in the relationship (or lack of relationship) between remedial math content and the content in credit-based math courses that students take later. Like most community colleges, the CUNY system of math placement exams, remedial math courses, and remedial exit exams requires that all degree-seeking community college students have or develop algebra skills that are aligned with and prepare them for the “college algebra—pre-calculus—calculus” sequence. This is the case even when those math courses are not relevant for students’ academic or career goals.[77] This means that large numbers of community college students at CUNY (including CTI students) who ultimately enroll in credit-based courses other than college algebra (especially statistics) have not studied remedial math content that would prepare them well for those courses. One alternative to this system would be to create a functions-based remedial algebra course for students who are not entering math-intensive or STEM majors. This course could emphasize the study of functions in realistic and real settings, the intersection of algebra and statistics, and the use of graphing calculator and other technologies. Another alternative for these students would be a Statway-like course that blends remedial topics into a year-long statistics course. If any new remedial math pathways are introduced, it adds an advisement challenge because it is very important that each student determines their own appropriate pathway when they enroll as freshmen.[78] This challenge seems worth taking on, though, and could lead to stronger student outcomes in credit-based math courses.

Poor alignment of math content (and high stakes assessments linked to the content) can actually put students at a disadvantage at multiple junctures in their educational careers. The New York State Regents exams, the GED, the COMPASS math placement exams, remedial math courses, and introductory credit-based math courses all may have different notions about what mathematics students should know and be able to do. As a result, math courses at any of these stages—even very high-quality math courses—may not do a good job of preparing students for what comes next.

[77] An emerging exception to this is The New Community College Initiative at CUNY (NCC). This seventh CUNY community college is scheduled to open in the fall of 2012. The NCC faculty and administration plan to require all students to complete a credit-based statistics course in their first year of study. Those who enter the college with significant math weaknesses will not do any zero-credit remedial courses, but will do the statistics course over two semesters with the basic skills work incorporated when it is needed. Students in the human services, liberal arts, and other non-STEM courses of study will complete their math requirement with the statistics course. Students in STEM and business fields will continue after statistics along the traditional sequence of courses that lead to calculus. There certainly are similarities to Statway here for non-STEM students, but the NCC plan differs from Statway in that all students (even STEM students) will begin by completing a statistics course.

[78] At CUNY community colleges, new students are asked to declare their course of study when they apply and enroll, even when they do not have good information or understanding of the academic and job implications of this decision. Associate’s Degree programs can include few elective courses, making it sometimes difficult for a student to change his/her course of study without having to take additional credits and potentially impact their long-term financial aid eligibility. If more than one remedial math pathway exists, a student who changes his/her mind and winds up changing pathways would have the same problem. Even if a new remedial math pathway is not developed, there continues to be a pressing need for high-quality, intensive academic and career advisement for students *in advance of* their first semester.

Boosting Instruction

A remedial course targeting a broad set of math learning goals will require more instructional time than a typical 45- or even a 60-hour course. CTI has demonstrated some very promising results in its 130-hour course, with more instruction in Phase Two for those who need it. Additional instructional time is not recommended here so that students can simply be drilled on more procedures through worksheets, computer software, tutoring labs, or other forms of "supplemental" instruction.[79] More time is needed in the classroom with an instructor so that students can work on and discuss a rich mixture of activities that will develop their mathematical proficiency in broad ways. Students need time together to explore, justify, critique, question, conjecture, struggle, and generalize as they examine mathematical relationships and develop and deepen their understandings.

The instructional intensity of the CTI math course may appear extraordinary at first, but we should remember how many students do not pass traditional remedial courses and then repeat them at least once. The intensive Phase One CTI course also blends pre-algebra and algebra content, helping many students to complete two levels of math remediation in one semester.

Rather than considering increases in instructional intensity, though, most colleges are trying to reduce or compress hours spent in remedial math courses through "express" or "accelerated" instructional models. Express or other abridged courses presume that students only need a "brush up" in their skills, but we have found that the majority of students who place into remedial math (including those who place into the highest remedial math level) instead have very deep math weaknesses. If there are some students who only need a refresher course, computerized placement exams are poor instruments for detecting who those students are. We should also consider the negative consequences of express courses for students who have fundamental academic weaknesses and who do not succeed in them. The flurry of algorithms presented in this rapid-fire form of behaviorist teaching probably adds to students' confusion, frustration, and certainty that they cannot learn math. A better approach with very weak students, if there is support for a few days or weeks of instruction in advance of the main semester, would be to spend the time carefully establishing deep knowledge of a very small number of foundational topics.

"Accelerated" programs may not reduce instructional time, but compress it as a way of appealing to students who might otherwise be frustrated and impatient with their placement in remediation. As is the case in traditional courses, accelerated courses will lead to behaviorist teaching methods because the instructional intensity and packed syllabi generally do not allow for anything else. Because the teaching remains basically unchanged, it is unlikely that accelerated programs can achieve significantly better results. Moving towards more intensive, "balanced/constructivist" teaching methods can help to reduce students' frustration with math learning, and this may help them to stay connected and become more patient learners—something they will need in subsequent math and other college courses.

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[79] Tutors who do not have extensive training can unintentionally undermine "balanced/constructivist" pedagogy from the classroom. For this reason, CTI instructors do not send students wanting extra help to college tutoring labs where CTI pedagogy is unlikely to be reinforced. Instead, CTI instructors will spend extra time with students outside of class and also encourage students to form study groups. Study groups can develop quite naturally because of the constant student collaboration that goes on during class sessions. CTI does use a few of its own tutors, but only under very specific circumstances. A student may become a CTI math tutor after she has demonstrated excellent performance in the CTI course, when she is enthusiastic about the our teaching approaches, and when the instructor is confident that the student will be able to reinforce those teaching approaches. Tutors work with students alongside the instructor during class sessions. Colleges can spend significant resources on the vast array of tutoring arrangements (including physical space, computer hardware and software, hourly pay for tutors, pay for managers who must coordinate their work, etc.), and it is worth considering whether some or many of these resources might be better spent adding instructional time to remedial math courses.

Because the full CTI program requires students to make a full-time commitment, this raises the question of what can be done to serve students who cannot attend classes 25 hours per week. Part-time students who are placed into remedial math need an intensive course for the same reasons that full-time students do. In order to benefit from more intensive instruction, then, part-time students would likely need to devote an entire semester of study to this type of remedial math course.

Improving Student Assessment

Assessment of remedial math students generally occurs at three different points—when students take placement exams and are put into specific remedial math courses, during those courses, and at the end of the courses to determine if the students are ready to move on to what comes next. If more "balanced/constructivist" pedagogy is to take hold, improved assessments should be created at all of these points that reflect an extensive set of learning goals.

Computerized placement exams such as the COMPASS should not be used (or should not be used on their own) to determine course placement because a handful of response to multiple-choice questions reveal too little about students' mathematical ability—especially ability that goes beyond procedural knowledge. To improve placement, student enrollment should include brief "math interviews" by faculty members using a carefully-constructed set of prompts that are designed to gather information on the student's conceptual understanding, procedural fluency, reasoning, communication ability, as well as their recent math learning history, confidence as a math learner, or some other affective measures.[80] For colleges who will dismiss this suggestion as too resource-intensive, math interviews could be limited to the subgroup of students whose placement scores lay a certain number of points above or below the traditional "cut score".[81] It is important to be generous in constructing this range. Students falling below this range would receive an intensive remedial math course, students scoring above this range would be placed in a credit-bearing math course, and students falling within the range would receive a math interview where a well-informed placement determination can be made.

Math interviews would improve student placement in several ways. Interviews could catch students who do or do not belong in remedial math classes but whose COMPASS scores indicate the opposite. At colleges where there is more than one remedial math pathway, students could learn more in the math interview about those choices. At colleges where there is only one pathway, the math interview results could be used to do some mild ability grouping of students into "remedial" and "high-remedial" sections.[82] The interviews might also identify students for "ESOL high-remedial" sections specifically designed for students with strong math ability but who have little familiarity with the language and notation of U.S. math classrooms. Finally, interviews could help to identify students who need additional testing for potentially undiagnosed learning disabilities.

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[80] There is some evidence suggesting that placement decisions can be improved when exam scores are combined with students' self-reported description of their high school mathematics preparation. See *"Assessing Developmental Assessment in Community Colleges"* by Katherine Hughes and Judith Scott-Clayton, Community College Research Center Brief Number 50, page 2; February 2011.

[81] "Cut score" is a common way of describing the minimum passing scores on college placement exams.

[82] In my view, math interviews and any subsequent grouping should not be used to separate remedial math students into pre-algebra and algebra courses because a high-quality remedial math course needs to blend number and algebra content. Ability grouping should also not be used to limit "high-remedial" students to something less than a semester-long course. A good purpose for grouping some students into "high-remedial" sections is to give these students a course that is just as intensive but that has the advantage of pursing extensions that may not be possible in more typical remedial sections. In this scenario, all students who arrive at the college needing some basic skills math instruction would get an ambitious, intensive, semester-long course.

During any remedial math course, assessment of student learning should be aligned with the extensive set of learning goals described earlier. To provide timely information so that the instructor can optimize instruction and so that students can understand their progress towards these goals, assessments should take many forms. Several appropriate assessment types were described in an earlier section of this paper and should include discussion of student work in exams and quizzes, discussion of student homework, discussions and questioning relating to the development of new content, individual conferences, and also a variety of student presentations, portfolios, self-assessments, and projects.

Assessments that are designed to determine whether a student has been successful in a remedial course must be aligned with the learning goals, content, and pedagogical orientation of that course. Unfortunately, the COMPASS placement exams have been used at CUNY to determine if students may exit from math remediation. It is impossible to say whether the COMPASS exams are in fact aligned with the content of remedial math courses at CUNY (or at other colleges where they are used in this way) because meaningful information on the exams is not shared by the publisher. The simple fact that remedial course syllabi differ across the six CUNY community colleges when their students have faced the same high-stakes exit exams should strike alarm bells among assessment experts. The placement exams were never intended, and should never have been used, in this way. The central CUNY Office of Academic Affairs is now planning a change to this practice. The plan is to assemble a committee of mathematics faculty who will agree on system-wide content “standards” for remedial pre-algebra and algebra courses. Final examinations will then be developed that are aligned with the standards. This is an improvement because faculty members are the only appropriate designers of assessments for their courses. And even though the system of determining when students will exit from remediation is changing for the better at CUNY as a whole, a discontinuity between the remediation assessment apparatus and the CTI math course may continue.

Like students in traditional remedial math classes at CUNY, CTI students have needed to pass the COMPASS math exams in order to exit remediation. This has been true even though the COMPASS exams are not aligned with the learning goals or assessments within the CTI math course. This misalignment has had important effects on the development of the course. To the extent that we know something about the content of the COMPASS exams, we know that they emphasize procedural knowledge. This has put pressure on the CTI math course to include quite a lot of practice on procedures while we also worked hard to develop students’ mathematical proficiency in broader ways. And because the COMPASS exams are not course assessments, they include a mixture of topics that do not necessarily connect well with one another. This has challenged the coherence of the CTI math course as it has strained to weave together a wide variety of topics. Beginning in the spring of 2011, CUNY is moving to higher cut scores on the pre-algebra and algebra exams, and this will intensify the pressure to include more procedural topics and remove activities (such as our work with functions on the graphing calculator that students benefit from and enjoy) that do not seem to directly lead to higher COMPASS scores.[83]

The plan to replace the COMPASS exams with system-wide remedial exit examinations may not improve the alignment of the CTI math course and its exit exams. The system-wide exams will be based on content standards devised by CUNY mathematics faculty, and those standards could reflect traditional notions of remedial mathematics education including an abundance of content and a narrow emphasis on procedural knowledge.

The misalignment between the CTI math course and CUNY remedial exit assessments is a good example of what Stigler and Hiebert argued—that components of the educational system in which teaching takes

[83] The minimum passing scores on the COMPASS pre-algebra/algebra exams are scheduled to increase from 30/30 to 35/40.

place (in this case, the assessment apparatus) can pressure educators to maintain or return to traditional teaching practices.[84]

Investing in Faculty Development

Rethinking learning goals, content, instructional intensity, and assessment help to create the space in which more "balanced/constructivist" math teaching can emerge. The fundamental remaining task, then, is to invest in faculty development that will help instructors make substantial changes in their approach to curriculum and teaching.

Our ability in CTI to develop a cadre of math instructors who practice high-quality "balanced/constructivist" pedagogical approaches has depended on a number of factors: instructor openness to innovative teaching methods, an intensive induction period for new instructors, an interest by the math team in proceeding in a collective manner, pedagogical leadership, and an administration that supported the work through its experimental phase. And even though CTI and community college math departments have important differences, the characteristics that have facilitated professional development in CTI, with some modifications, can do the same for faculty teaching remedial courses in community college settings.

The three founding CTP math instructors shared pedagogical values that stemmed from curricula and staff development work we had done together in the CUNY Adult Literacy/GED Program.[85] After three semesters, additional instructors were selected when they demonstrated pedagogical skill and values similar to ours, and/or because they were open to learning the course curriculum and pedagogy as cooperating instructors.

Because community college search committees generally consider a mixture of teaching, research, and service in their hiring decisions, there can be a good deal of variability in faculty members' pedagogical expertise and interests. With this variability in mind, it may be prudent for faculty development to begin with a subset of remedial math instructors who can agree on some pedagogical values, or at least on some pedagogical questions they want to investigate together. Once formed, a small group will have an easier time meeting, planning, teaching, observing, reflecting, and revising their work than a large one. Attempting faculty development with an entire math department would be very expensive and limit the intensity of development activities. Working with large numbers of faculty might also be hampered by the reality that some faculty members will not believe anything can be gained from the effort.[86] One way of identifying an initial group of reform-minded faculty would be to invite an innovative practitioner (from inside or outside the college) to host a discussion on "balanced/constructivist" pedagogy, on the research relating to how students learn mathematics, or on another topic relevant to instructional reform.

Focusing initially on a subset of instructors—a remedial math "reform team"—does not mean giving up on the idea of department-wide change. As the team begins to uncover promising practices, these

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[84] The Teaching Gap, page 99.

[85] The pedagogical values elaborated in *More Than Rules* would also apply to several pre-GED and GED math curricula used in the CUNY Adult Literacy/GED Program sites in the four years before CTP began. I wrote those curricula, and the other two CTP founding instructors (GED math teachers at the time) knew them well after using and adapting them in their own classrooms. These instructors joined the first CTP math class as cooperating teachers with a fairly good idea of the kinds of student understanding we hoped to cultivate, even if we were less certain of the precise content and activities we would eventually develop for the course.

[86] Some instructors may believe that low success rates in remedial math courses are primarily due to students' academic or other weaknesses and that those weaknesses are largely impervious to different teaching approaches. Or, some instructors may believe that there is one evident, sensible way to teach most math concepts. In these instances, there will be little enthusiasm for investigating new approaches.

practices must be shared with department and college leaders. Writing about and presenting this work more widely is also very important because the writing process helps members to clarify their thinking about what has been learned and what questions and challenges remain.[87]

The CTI professional development model has two important components. New instructors receive intensive, classroom-based training from a more experienced instructor for one semester, and continuing CTI instructors join together in a collaboration that is designed to make incremental improvements in their teaching over the long term.

An intensive period of classroom observation has been a vital component of instructor induction in CTI. Cooperating instructors spend one semester alongside a more experienced instructor to study and discuss the curriculum documents, observe the teaching of the lessons, assist students during class sessions, and gradually take on responsibility for leading individual pieces of the curriculum. The induction semester is vital precisely because CTI teaching practices break so dramatically from traditional practices. In particular, the cooperating instructors need time to recognize, practice, and get feedback on a range of classroom facilitation skills that are very uncommon and that cannot be fully articulated in any curriculum document. Even with nearly 200 pages of detailed instructor notes in the Phase One curriculum, it is highly unlikely that a newcomer could pick up the document and use it in the way it is intended without also witnessing several weeks of our teaching. After the semester, cooperating instructors become full instructors of their own classes.

After participating in the induction semester, CTI instructors agree to be a part of a group which collectively decides how the curriculum will unfold each semester in all CTI classrooms.[88] The group meets to decide when to change the approaches to a topic, the goals for class discussions, or even to revise individual examples in discussions, activities, and assessments. This collective approach puts an end to the isolation that is so common among instructors. Group conversations about how to improve the activities and meet our extensive set of learning goals are focused and productive because the instructors can share experiences after using almost identical materials with different groups of students. This intensive focus on continuously improving a shared set of activities bears some resemblance to the Japanese “lesson study” approach to long-term, incremental, professional development.

In community college remedial mathematics programs, it is rare that classroom observations, and discussions about them, are a part of faculty development projects. This can change. A fledgling reform team would benefit enormously from joining together for a semester to teach a single class of students. Depending on the pedagogical experience and interests of team members, a semester of observation/co-instruction could be geared towards learning more about one another’s classroom practices (by dividing up the semester so each teaches for 2-3 weeks), to carefully observe and discuss the teaching of a faculty

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[87] Reform team members should develop their skills as teacher-researchers. This should include writing about students’ precise mathematical understandings and misunderstandings and the pedagogies and classroom activities that seem to be affective in addressing students’ strengths and weaknesses. Team members should not be dissuaded from doing this type of practitioner research by other researchers who seem to only value randomized, controlled studies. In the paper “*Improving the Performance of the Education Sector: The Valuable, Challenging, and Limited Role of Random Assignment Evaluations”*, Murmane and Nelson connect educational research to the work of Atul Gawande who has written about the importance of innovation and sharing among doctors. Murmane and Nelson point out that Gawande’s account of innovation in cystic fibrosis treatments—specifically, the way that doctors shared their patient observations with one another—was enormously successful at improving treatments even though “systematic evaluation research seems to have been modest”. Their argument is that randomized control trials may be overvalued in comparison to other types of educational research, and we therefore may be missing important opportunities to improve teaching and learning. Murnane and Nelson’s article was published in the National Bureau of Economic Research; December 2005.

[88] For more on the “living curriculum”, see *More Than Rules* pages 33-34.

member who has particular pedagogical expertise (more like the CTI induction semester), or to investigate one or more pedagogical questions the team creates together. Extensive time should be built in before, during, and after the semester for the team to plan and reflect on this work. As the understanding and practices of reform team members evolves, one of the best ways to share this with other faculty is to open subsequent classes to observers.

Creating, experimenting with, reflecting on, and revising a set of shared activities across multiple classrooms would also be very important for a community college remedial math reform team. The goal here would be to focus conversations about what is working and what is not working by drawing on as many shared experiences as possible.

In order to design and carry out professional development in CTP and CTI, pedagogical leadership has been needed. I have been a curriculum and professional development leader in the math team since its inception. Especially in the early semesters, it was important that I was able to identify potential instructors, draft lessons that embody our teaching and learning principles, analyze college remedial math syllabi and information that is publicly available on the COMPASS exams, observe instructors to provide feedback on their classroom practice, and organize math team meetings. It was important that I taught the classes myself multiple times (to have my own experiences with students, and to host cooperating teachers), and it was also important that I was free from teaching over other semesters so that I could focus my attention on drafting and revising new lessons for the course when it grew from 42 hours, to 72 hours, and then to 130 hours. Gradually, other members of the team have developed very strong skills in these areas, and this is reflected in their increased roles in the math team and in the program as a whole.

A community college reform team will also need one or more leaders who have a range of pedagogical expertise and curriculum-writing ability. When this expertise does not exist in the department, the team will need to look to writings and experts from the outside to assist in their work. Colleges of education at nearby universities could be potential sources of this expertise. Presently, I do not see colleges of education routinely partnering with community colleges for this purpose, even though mathematics teacher educators spend their time preparing middle and high school teachers to effectively teach content that closely resembles what adult students are struggling with in community college remedial courses. There may be challenges in establishing these types of cross-institutional partnerships, but it would be worth the effort if community college faculty could tap into the expertise of our math teacher educator colleagues.

Administrators in the CUNY Central Office of Academic Affairs (OAA) and at several CUNY campuses provided critical financial and other support during the development of CTP and CTI. Because CTP began as a small pre-college program and not in an existing college math department, we were able to experiment with content, the sequence of topics, internal assessments, and other features of the math course. In the first few semesters of this work when instructional intensity was very limited, we saw promising developments in student learning but they were not yet passing the college placement exams at high rates. Rather than closing down the work because it did not instantly show remarkable pass rates, OAA administrators encouraged us to do what we thought was needed to help students to be successful. They supported the increases in instruction and advisement we requested, and this led to more encouraging student outcomes in the fall 2008 and spring 2009 cohorts (described in *More Than Rules*). It was after more than two years of support for our early CTP efforts, then, that OAA administrators felt there were enough promising results to establish and fund CTI in the fall 2009 semester.

A community college reform team will also need support from college and department leaders. Resources will be needed for release time so that faculty may observe, co-instruct, plan, reflect, and participate in professional development activities. Support will be needed to share findings within and outside of the department and college. As was the case for CTP and CTI, there must be a commitment to support the effort over years and not just one or two semesters. Having this kind of patience can be difficult when there is so much pressure around the country to find a quick fix for remedial math education. This patience must withstand outcome measures that may not show improvement or may even dip in the first few semesters as faculty engage in substantial experimentation. College and department leaders should encourage talented faculty members to work in the reform team by pledging to reward this service as highly as any other professional activity in promotion and tenure decisions. Finally, support should include flexibility so that team members can deviate from departmental syllabi, exams, typical hours of instruction, or make other changes as a part of their pedagogical experimentation.

And while patience is needed among administrators in the early stages of innovative work, another kind of patience is needed when innovative work starts to show real promise. Largely because of the COMPASS exam scores for CTI students, the CUNY administration hopes to replicate this work more widely at the University. This planned expansion may pose a challenge for the CTI math instructor team. In the early semesters of this work when there were relatively few instructors, it was possible (and vital) for the instructor team to gather in person to discuss and make changes in the curriculum every semester. Now that there are instructors at four CUNY campuses (with plans to expand to three more), and these instructors have different full-time teaching schedules and academic calendars, it is more difficult for the team to collaborate in person precisely when changes to the CUNY remedial mathematics testing apparatus are on the horizon. At the same time, increasing energy is being directed towards finding and inducting more cooperating instructors who will become able to teach CTI math classes in the new campus programs. What we are learning is that it can be a challenge to simultaneously expand and carefully improve an innovative math program.

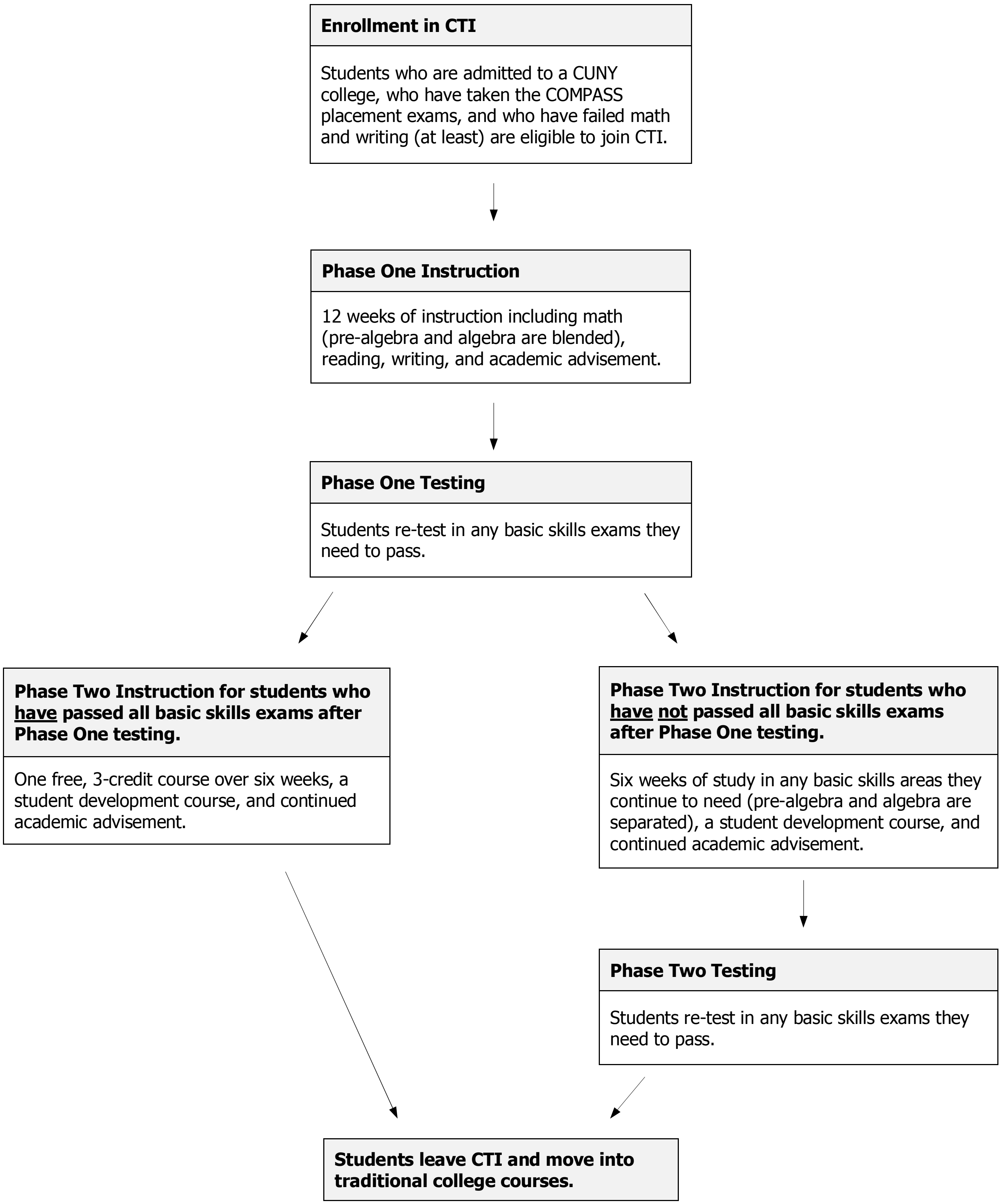
The potential of faculty development rests in the expectation that as we gain new insights into student learning, we will attempt to carry these new practices into our classrooms. As community college faculty reform teams do their work and identify the benefits of more “balanced/constructivist” teaching methods, they will need fellow faculty members, department leaders, and college leaders to support more widespread adoption of these practices. Faculty and administrators may tend to shy away from addressing teaching practice directly, perhaps because it seems so complex and difficult to change. It is true that buying new math software or adding some tutors are much easier to accomplish by comparison, but those changes rarely address the fundamental issues facing instructors and students. I believe that the path to significant improvement in community college mathematics education must include fresh thinking about teaching methods. And while I am very aware of the many challenges that are involved in a shift towards more “balanced/constructivist” teaching, I remain hopeful that the recent surge of interest in community college student success and especially in remedial education can translate into more experimentation in this important direction.

Appendix A

Comparing Program Components in the College Transition Program and College Transition Initiative

	College Transition Program *Initial Model*	College Transition Program *"Intensive" Model*	College Transition Initiative
Program semesters	Spring 2007 through summer 2008	Fall 2008 and spring 2009	Fall 2009 forward
Student eligibility	Could be co-enrolled in GED classes or a recent graduate but could not have taken college placement exams.	Must be a GED graduate but could not have enrolled in college or have taken college placement tests.	Must be a GED (or h.s.) graduate who has been admitted to a CUNY college, tested, and failed writing and math (at least).
Weekly program hours for students	3 or 6, depending on whether the student enrolled in the math course, the reading/writing course, or both courses.	13 or 14	25
Program duration	14 weeks	14 weeks	18 weeks divided into 12-week Phase One and 6-week Phase Two.
Total hours of math instruction	42	72	130 in Phase One and, if needed, 35 to 50 additional hours in Phase Two.
Learning community?	No	Yes	Yes
Academic advisement	Trace	A part-time Advisor provided 1 hour per week of group advisement plus individual advisement and counseling.	A ¾-time Advisor is devoted to two classes, providing group and individual advisement and counseling.
Cost to the student	$0	$0	$75
Opportunities to take basic skills exams	1	1	2
Status of instructional staff	Part-time	Part-time	Full-time

Appendix B



Enrollment in CTI
Students who are admitted to a CUNY college, who have taken the COMPASS placement exams, and who have failed math and writing (at least) are eligible to join CTI.
Phase One Instruction
12 weeks of instruction including math (pre-algebra and algebra are blended), reading, writing, and academic advisement.
Phase One Testing
Students re-test in any basic skills exams they need to pass.
Phase Two Instruction for students who have passed all basic skills exams after Phase One testing.
One free, 3-credit course over six weeks, a student development course, and continued academic advisement.
Phase Two Instruction for students who have not passed all basic skills exams after Phase One testing.
Six weeks of study in any basic skills areas they continue to need (pre-algebra and algebra are separated), a student development course, and continued academic advisement.
Phase Two Testing
Students re-test in any basic skills exams they need to pass.
Students leave CTI and move into traditional college courses.

Appendix C

The following averages may help other researchers compare CTI student outcomes to other student populations. This table includes 111 different students who completed CTI Session One and re-took at least one COMPASS math exam at least one time. For students who took a math exam after Phase One and after Phase Two, their best score is included.

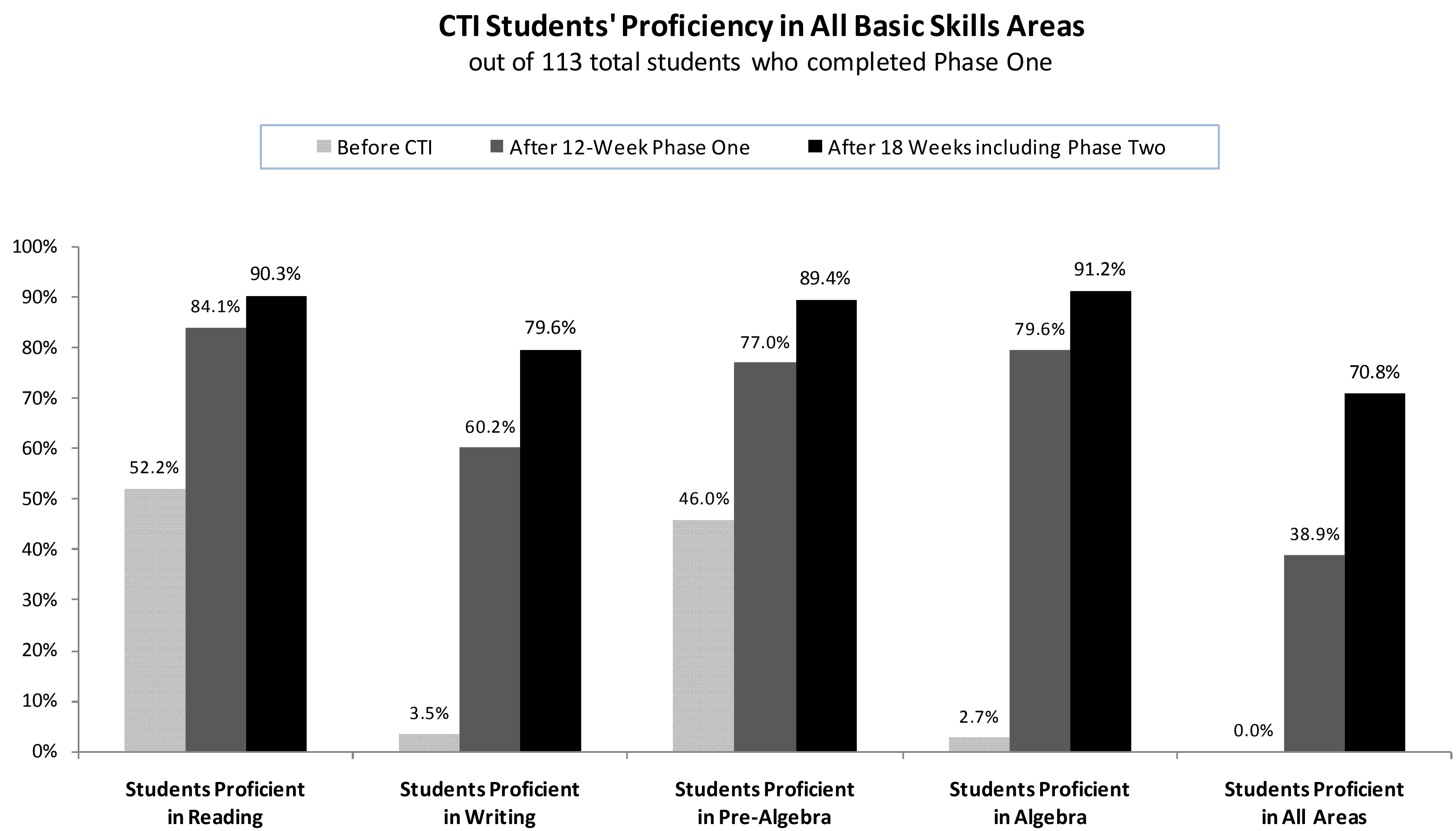
COMPASS Score Detail for CTI Students, Fall 2009 and Spring 2010

	Mean score before CTI	Standard Deviation	Mean score after CTI	Standard Deviation
Pre-algebra score for all students needing pre-algebra at the beginning of CTI (n = 61)	23.7	3.5	41.7	15.0
Pre-algebra score for students who ultimately passed pre-algebra during CTI (n = 50)	24.4	3.3	45.4	14.0
Pre-algebra score for students who had not passed pre-algebra by the end of CTI (n = 11)	20.4	2.5	25.1	2.9
Algebra score for all students needing algebra at the beginning of CTI (n = 109)	19.8	4.1	41.3	11.5
Algebra score for students who ultimately passed algebra during CTI (n = 101)	19.8	4.0	42.9	10.5
Algebra score for students who had not passed algebra by the end of CTI (n = 8)	19.8	4.8	22.4	4.2

Appendix D

The graph on the following page shows student proficiency in all four basic skills areas at the beginning of the CTI semester, after Phase One, and after Phase Two. The data refers to the 113 students who completed Phase One because they re-tested on any basic skills exams.[89]

[89] For three students who completed Phase One instruction but did not re-take all of the basic skills exams they needed, the graph and subsequent table appropriately show the students remained "not proficient" in a basic skills area when a test was not re-taken. This is indistinguishable in the data from students who re-took and failed the exams. One student did not re-take the COMPASS math exams she needed despite completing Phase One instruction and re-testing in writing. The math pass rates in this graph appear slightly lower than they did in the bar graph on page 12 because that earlier graph did not count the one student who did not re-test on the math exams. And, as was noted earlier, four students entered CTI too late to take the official ACT writing exam and therefore received unofficial writing scores. The basic skills needs indicated by the unofficial scores are included in the graph and table that follows.


CTI Students' Proficiency in All Basic Skills Areas
out of 113 total students who completed Phase One
Before CTI
After 12-Week Phase One
After 18 Weeks including Phase Two
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
52.2%
84.1%
90.3%
3.5%
60.2%
79.6%
46.0%
77.0%
89.4%
2.7%
79.6%
91.2%
0.0%
38.9%
70.8%
Students Proficient in Reading
Students Proficient in Writing
Students Proficient in Pre-Algebra
Students Proficient in Algebra
Students Proficient in All Areas

The chart below shows the distribution of basic skills needs for students at the beginning of the CTI semester, after Phase One, and after Phase Two. The data again refer to outcomes for 113 students because they re-tested on any basic skills exams.

Entering and Exiting Basic Skills Needs for CTI Students			
	Before CTI	After 12 weeks of CTI	After 18 weeks of CTI
Students who needed to pass all four basic skills exams	30	3	3
Students who needed to pass three basic skills exams	50	10	3
Students who needed to pass two basic skills exams	31	14	8
Students who needed to pass one basic skills exam	2	42	19
Students proficient in all basic skills areas	0	44	80
Average number of exams needed per student	3.0	1.0	0.5

The graph and table demonstrate a vast reduction in students' overall need for remediation. Before the CTI semester, 111 out of 113 students (98.2%) had two or more basic skills needs. After eighteen weeks of CTI, only fourteen students (12.4%) had two or more basic skills needs, and eight of those had not taken full advantage of CTI instruction because they did not start or complete Phase Two.

The rate that CTI students reached proficiency in all areas can be considered in relation to the typical performance of GED enrollees at CUNY. A much larger share of CTI *starters* reached proficiency in all basic skills areas after one semester of study (80 out of 141 or 57%) than was found for a large pool of GED graduates at CUNY after one year of study (just 33%).[90]

[90] The 33% statistic applies to GED graduates at CUNY in the 2001-2002, 2004-2005, and 2006-2007 cohorts and was taken from the report "*College Readiness of New York City's GED Recipients*" created by the CUNY Office of Institutional Research and Assessment, 2008, page 22. The CTI group and the larger GED graduate group have differences and so we should not draw hasty conclusions from this comparison. Firstly, while dominated by GED graduates, the CTI pool includes a small group of high school graduates. CTI students also attend classes full-time and so we would expect this group to progress more quickly than a broad pool of GED graduates that includes part-time students. On the other hand, it has already been demonstrated that CTI students entered college with more basic skills needs and lower GED scores than typical GED enrollees. With these cautions in mind, the rate that CTI starters reach full proficiency is still very encouraging.

Appendix E

College Outcomes for CTP Students Entering CUNY Associate Degree Programs in Spring 2009 and Fall 2009[91]			
	CTP Alumni[92] *n* = 36	All GED Grads[93] *n* = 2,164	All Freshmen[94] *n* = 21,895
First Semester GPA	2.78	2.14	2.19
	CTP Alumni Enrolling Full-Time *n* = 27	All GED Grads Enrolling Full-Time *n* = 1,747	All Freshmen Enrolling Full-Time *n* = 19,328
First Semester Credits Attempted	12.04	7.44	8.61
First Semester Credits Earned	11.04	4.85	6.40
Ratio of Credits Earned to Attempted	91.7%	65.2%	74.3%
Persistence to a Second Semester	96.3%	73.1%	84.2%

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[91] Data collected by CUNY Collaborative Programs Research and Evaluation.
[92] These students attended the CUNY College Transition Program (CTP) in the fall 2008 or spring 2009 semester and entered CUNY in the spring 2009 or fall 2009 semesters. CTP was described in *More Than Rules* and was the predecessor program to CTI. Only students enrolled in CUNY degree programs (and not most certificate programs) are included in this data.
[93] Data include first-time freshmen entering in the fall 2008 semester.
[94] Data include first-time freshmen entering in the fall 2008 semester.